

# The WiggleZ Dark Energy Survey: testing the cosmological model with baryon acoustic oscillations at $z = 0.6$

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## ABSTRACT

We measure the imprint of baryon acoustic oscillations (BAOs) in the galaxy clustering pattern at the highest redshift achieved to date,  $z = 0.6$ , using the distribution of  $N = 132\,509$  emission-line galaxies in the WiggleZ Dark Energy Survey. We quantify BAOs using three statistics: the galaxy correlation function, power spectrum and the band-filtered estimator introduced by Xu et al. The results are mutually consistent, corresponding to a 4.0 per cent measurement of the cosmic distance–redshift relation at  $z = 0.6$  [in terms of the acoustic parameter ‘ $A(z)$ ’ introduced by Eisenstein et al., we find  $A(z = 0.6) = 0.452 \pm 0.018$ ]. Both BAOs and power spectrum shape information contribute towards these constraints. The statistical significance of the detection of the acoustic peak in the correlation function, relative to a wiggle-free model, is  $3.2\sigma$ . The ratios of our distance measurements to those obtained using BAOs in the distribution of luminous red galaxies at redshifts  $z = 0.2$  and  $0.35$  are consistent with a flat  $\Lambda$  cold dark matter model that also provides a good fit to the pattern of observed fluctuations in the cosmic microwave background radiation. The addition of the current WiggleZ data results in a  $\approx 30$  per cent improvement in the measurement accuracy of a constant equation of state,  $w$ , using BAO data alone. Based solely on geometric BAO distance ratios, accelerating expansion ( $w < -1/3$ ) is required with a probability of 99.8 per cent, providing a consistency check of conclusions based on supernovae observations. Further improvements in cosmological constraints will result when the WiggleZ survey data set is complete.

**Key words:** surveys – cosmological parameters – dark energy – large-scale structure of Universe.

## 1 INTRODUCTION

The measurement of baryon acoustic oscillations (BAOs) in the large-scale clustering pattern of galaxies has rapidly become one

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of the most important observational pillars of the cosmological model. BAOs correspond to a preferred length-scale imprinted in the distribution of photons and baryons by the propagation of sound waves in the relativistic plasma of the early Universe (Peebles & Yu 1970; Sunyaev & Zeldovitch 1970; Bond & Efstathiou 1984; Holtzman 1989; Hu & Sugiyama 1996; Eisenstein & Hu 1998). A full account of the early-universe physics is provided by Bashinsky & Bertschinger (2001, 2002). In a simple intuitive description of the effect we can imagine an overdensity in the primordial dark matter distribution creating an overpressure in the tightly coupled photon–baryon fluid and launching a spherical compression wave. At redshift  $z \approx 1000$ , there is a precipitous decrease in sound speed due to recombination to a neutral gas and decoupling of the photon–baryon fluid. The photons stream away and can be mapped as the cosmic microwave background (CMB) radiation; the spherical shell of compressed baryonic matter is frozen in place. The overdense shell, together with the initial central perturbation, seeds the later formation of galaxies and imprints a preferred scale into the galaxy distribution equal to the sound horizon at the baryon drag epoch. Given that baryonic matter is secondary to cold dark matter (CDM) in the clustering pattern, the amplitude of the effect is much smaller than the acoustic peak structure in the CMB.

The measurement of BAOs in the pattern of late-time galaxy clustering provides a compelling validation of the standard picture that large-scale structure (LSS) in today’s Universe arises through the gravitational amplification of perturbations seeded at early times. The small amplitude of the imprint of BAOs in the galaxy distribution is a demonstration that the bulk of matter consists of non-baryonic dark matter that does not couple to the relativistic plasma before recombination. Furthermore, the preferred length-scale – the sound horizon at the baryon drag epoch – may be predicted very accurately by measurements of the CMB which yield the physical matter and baryon densities that control the sound speed, expansion rate and recombination time: the latest determination is  $153.3 \pm 2.0$  Mpc (Komatsu et al. 2009). Therefore, the imprint of BAOs provides a standard cosmological ruler that can map out the cosmic expansion history and provide precise and robust constraints on the nature of the ‘dark energy’ that is apparently dominating the current cosmic dynamics (Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003). In principle, the standard ruler may be applied in both the tangential and radial directions of a galaxy survey, yielding measures of the angular diameter distance and Hubble parameter as a function of redshift.

The large scale and small amplitude of the BAOs imprinted in the galaxy distribution imply that galaxy redshift surveys mapping cosmic volumes of order  $1 \text{ Gpc}^3$  with of order  $10^5$  galaxies are required to ensure a robust detection (Tegmark 1997; Blake & Glazebrook 2003; Blake et al. 2006). Gathering such a sample represents a formidable observational challenge typically necessitating hundreds of nights of telescope time over several years. The leading such spectroscopic data set in existence is the Sloan Digital Sky Survey (SDSS), which covers  $8000 \text{ deg}^2$  of sky containing a ‘main’  $r$ -band selected sample of  $10^6$  galaxies with median redshift  $z \approx 0.1$ , and a luminous red galaxy (LRG) extension consisting of  $10^5$  galaxies but covering a significantly greater cosmic volume with median redshift  $z \approx 0.35$ . Eisenstein et al. (2005) reported a convincing BAO detection in the two-point correlation function of the SDSS Third Data Release (DR3) LRG sample at  $z = 0.35$ , demonstrating that this standard-ruler measurement was self-consistent with the cosmological model established from CMB observations and yielding new, tighter constraints on cosmological parameters such as the spatial curvature. Percival et al. (2010) undertook a power-

spectrum analysis of the SDSS DR7 data set, considering both the main and LRG samples, and constrained the distance–redshift relation at both  $z = 0.2$  and  $0.35$  with  $\sim 3$  per cent accuracy in units of the standard-ruler scale. Other studies of the SDSS LRG sample, producing broadly similar conclusions, have been performed by Huetsi (2006), Percival et al. (2007), Sanchez et al. (2009) and Kazin et al. (2010a). Some analyses have attempted to separate the tangential and radial BAO signatures in the LRG data set, albeit with lower statistical significance (Gaztanaga, Cabre & Hui 2009; Kazin et al. 2010b). These studies built on earlier hints of BAOs reported by the Two-degree Field Galaxy Redshift Survey (Cole et al. 2005) and combinations of smaller data sets (Miller, Nichol & Batuski 2001). A measurement of the baryon acoustic peak within the 6-degree field Galaxy Survey was recently reported at low redshift  $z = 0.1$  by Beutler et al. (2011).

This ambitious observational programme to map out the cosmic expansion history with BAOs has prompted serious theoretical scrutiny of the accuracy with which we can model the BAO signature and the likely amplitude of systematic errors in the measurement. The pattern of clustering laid down in the high-redshift Universe is potentially subject to modulation by the non-linear scale-dependent growth of structure, by the distortions apparent when the signal is observed in redshift space, and by the bias with which galaxies trace the underlying network of matter fluctuations. In this context, the fact that the BAOs are imprinted on large, linear and quasi-linear scales of the clustering pattern implies that non-linear BAO distortions are relatively accessible to modelling via perturbation theory or numerical  $N$ -body simulations (Eisenstein, Seo & White 2007; Crocce & Scoccimarro 2008; Matsubara 2008). The leading-order effect is a ‘damping’ of the sharpness of the acoustic feature due to the differential motion of pairs of tracers separated by 150 Mpc driven by bulk flows of matter. Effects due to galaxy formation and bias are confined to significantly smaller scales and are not expected to cause significant acoustic peak shifts. Although the non-linear damping of BAOs reduces to some extent the accuracy with which the standard ruler can be applied, the overall picture remains that BAOs provide a robust probe of the cosmological model free of serious systematic error. The principle challenge lies in executing the formidable galaxy redshift surveys needed to exploit the technique.

In particular, the present ambition is to extend the relatively low-redshift BAO measurements provided by the SDSS data set to the intermediate- and high-redshift Universe. Higher redshift observations serve to further test the cosmological model over the full range of epochs for which dark energy apparently dominates the cosmic dynamics, can probe greater cosmic volumes and therefore yield more accurate BAO measurements, and are less susceptible to the non-linear effects which damp the sharpness of the acoustic signature at low redshift and may induce low-amplitude systematic errors. Currently, intermediate redshifts have only been probed by photometric-redshift surveys which have limited statistical precision (Blake et al. 2007; Padmanabhan et al. 2007).

The WiggleZ Dark Energy Survey at the Australian Astronomical Observatory (Drinkwater et al. 2010) was designed to provide the next-generation spectroscopic BAO data set following the SDSS, extending the distance-scale measurements across the intermediate-redshift range up to  $z = 0.9$  with a precision of mapping the acoustic scale comparable to the SDSS LRG sample. The survey, which began in 2006 August, completed observations in 2011 January and has obtained of order 200 000 redshifts for UV-bright emission-line galaxies covering of order  $1000 \text{ deg}^2$  of equatorial sky. Analysis of the full data set is ongoing. In this paper we report intermediate

results for a subset of the WiggleZ sample with effective redshift  $z = 0.6$ .

BAOs are a signature present in the two-point clustering of galaxies. In this paper, we analyse this signature using a variety of techniques: the two-point correlation function, the power spectrum and the band-filtered estimator recently proposed by Xu et al. (2010) which amounts to a band-filtered correlation function. Quantifying the BAO measurement using this range of techniques increases the robustness of our results and gives us a sense of the amplitude of systematic errors induced by our current methodologies. Using each of these techniques we measure the angle-averaged clustering statistic, making no attempt to separate the tangential and radial components of the signal. Therefore, we measure the ‘dilation scale’ distance  $D_V(z)$  introduced by Eisenstein et al. (2005) which consists of two parts physical angular diameter distance,  $D_A(z)$ , and one part radial proper distance,  $cz/H(z)$ :

$$D_V(z) = \left[ (1+z)^2 D_A(z)^2 \frac{cz}{H(z)} \right]^{1/3}. \quad (1)$$

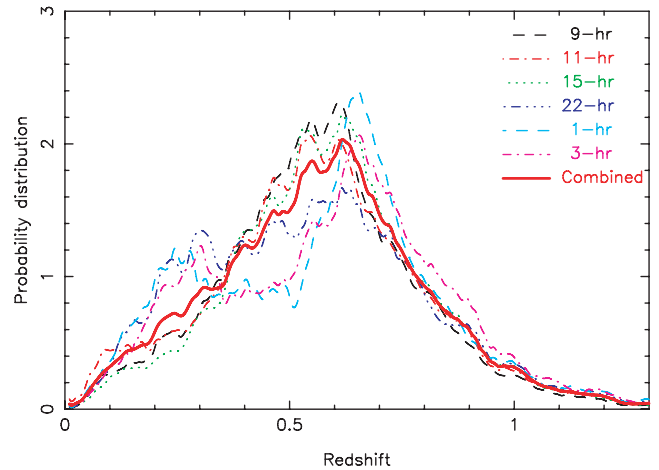
This distance measure reflects the relative importance of the tangential and radial modes in the angle-averaged BAO measurement (Padmanabhan & White 2008), and reduces to proper distance in the low-redshift limit. Given that a measurement of  $D_V(z)$  is correlated with the physical matter density  $\Omega_m h^2$  which controls the standard-ruler scale, we extract other distilled parameters which are far less significantly correlated with  $\Omega_m h^2$ : the acoustic parameter  $A(z)$  as introduced by Eisenstein et al. (2005); the ratio  $d_z = r_s(z_d)/D_V(z)$ , which quantifies the distance scale in units of the sound horizon at the baryon drag epoch,  $r_s(z_d)$ ; and  $1/R_z$  which is the ratio between  $D_V(z)$  and the distance to the CMB last-scattering surface.

The structure of this paper is as follows. The WiggleZ data sample is introduced in Section 2, and we then present our measurements of the galaxy correlation function, power spectrum and band-filtered correlation function in Sections 3, 4 and 5, respectively. The results of these different methodologies are compared in Section 6. In Section 7 we state our measurements of the BAO distance scale at  $z = 0.6$  using various distilled parameters, and combine our result with other cosmological data sets in Section 8. Throughout this paper, we assume a fiducial cosmological model which is a flat  $\Lambda$ CDM Universe with matter density parameter  $\Omega_m = 0.27$ , baryon fraction  $\Omega_b/\Omega_m = 0.166$ , Hubble parameter  $h = 0.71$ , primordial index of scalar perturbations  $n_s = 0.96$  and redshift-zero normalization  $\sigma_8 = 0.8$ . This fiducial model is used for some of the intermediate steps in our analysis, but our final cosmological constraints are, to first order at least, independent of the choice of fiducial model.

## 2 DATA

The WiggleZ Dark Energy Survey at the Anglo Australian Telescope (Drinkwater et al. 2010) is a large-scale galaxy redshift survey of bright emission-line galaxies mapping a cosmic volume of order  $1 \text{ Gpc}^3$  over the redshift interval  $z < 1$ . The survey has obtained of order 200 000 redshifts for UV-selected galaxies covering of order  $1000 \text{ deg}^2$  of equatorial sky. In this paper we analyse the subset of the WiggleZ sample assembled up to the end of the 10A semester (2010 May). We include data from six survey regions in the redshift range  $0.3 < z < 0.9$  – the 9-, 11-, 15-, 22-, 1- and 3-h regions – which together constitute a total sample of  $N = 132\,509$  galaxies. The redshift probability distributions of the galaxies in each region are shown in Fig. 1.

The selection function for each survey region was determined using the methods described by Blake et al. (2010) which model



**Figure 1.** The probability distribution of galaxy redshifts in each of the WiggleZ regions used in our clustering analysis, together with the combined distribution. Differences between individual regions result from variations in the galaxy colour selection criteria depending on the available optical imaging (Drinkwater et al. 2010).

effects due to the survey boundaries, incompleteness in the parent UV and optical catalogues, incompleteness in the spectroscopic follow-up, systematic variations in the spectroscopic redshift completeness across the AAOmega spectrograph and variations of the galaxy redshift distribution with angular position. The modelling process produces a series of Monte Carlo random realizations of the angle/redshift catalogue in each region, which are used in the correlation function estimation. By stacking together a very large number of these random realizations, we deduced the 3D window function grid used for power spectrum estimation.

## 3 CORRELATION FUNCTION

### 3.1 Measurements

The two-point correlation function is a common method for quantifying the clustering of a population of galaxies, in which the distribution of pair separations in the data set is compared to that within random, unclustered catalogues possessing the same selection function (Peebles 1980). In the context of measuring BAOs, the correlation function has the advantage that the expected signal of a preferred clustering scale is confined to a single, narrow range of separations around  $105 h^{-1} \text{ Mpc}$ . Furthermore, small-scale non-linear effects, such as the distribution of galaxies within dark matter haloes, do not influence the correlation function on these large scales. One disadvantage of this statistic is that measurements of the large-scale correlation function are prone to systematic error because they are very sensitive to the unknown mean density of the galaxy population. However, such ‘integral constraint’ effects result in a roughly constant offset in the large-scale correlation function, which does not introduce a preferred scale that could mimic the BAO signature.

In order to estimate the correlation function of each WiggleZ survey region, we first placed the angle/redshift catalogues for the data and random sets on a grid of comoving coordinates, assuming a flat  $\Lambda$ CDM model with matter density  $\Omega_m = 0.27$ . We then measured the redshift-space two-point correlation function  $\xi(s)$  for

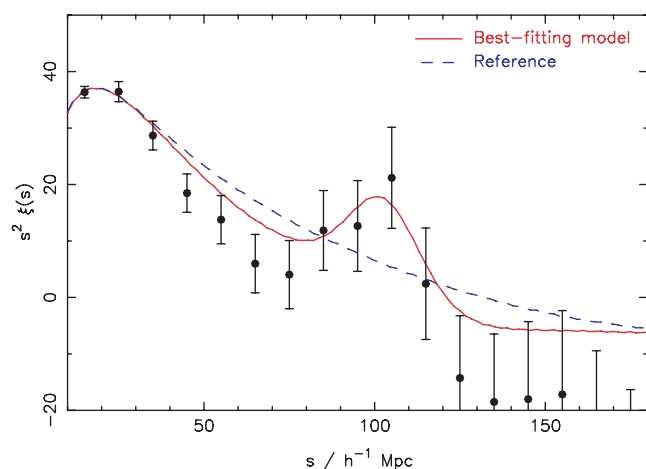
each region using the Landy & Szalay (1993) estimator:

$$\xi(s) = \frac{DD(s) - 2DR(s) + RR(s)}{RR(s)}, \quad (2)$$

where  $DD(s)$ ,  $DR(s)$  and  $RR(s)$  are the data–data, data–random and random–random weighted pair counts in separation bin  $s$ , each random catalogue containing the same number of galaxies as the real data set. In the construction of the pair counts, each data or random galaxy  $i$  is assigned a weight  $w_i = 1/(1 + n_i P_0)$ , where  $n_i$  is the survey number density (in  $h^3 \text{ Mpc}^{-3}$ ) at the location of the  $i$ th galaxy, and  $P_0 = 5000 h^{-3} \text{ Mpc}^3$  is a characteristic power spectrum amplitude at the scales of interest. The survey number density distribution is established by averaging over a large ensemble of random catalogues. The  $DR$  and  $RR$  pair counts are determined by averaging over 10 random catalogues. We measured the correlation function in 17 separation bins of width  $10 h^{-1} \text{ Mpc}$  between 10 and  $180 h^{-1} \text{ Mpc}$ , and determined the covariance matrix of this measurement using lognormal survey realizations as described below. We combined the correlation function measurements in each bin for the different survey regions using inverse-variance weighting of each measurement (we note that this procedure produces an almost identical result to combining the individual pair counts).

The combined correlation function is plotted in Fig. 2 and shows clear evidence for the baryon acoustic peak at separation  $\sim 105 h^{-1} \text{ Mpc}$ . The effective redshift  $z_{\text{eff}}$  of the correlation function measurement is the weighted mean redshift of the galaxy pairs entering the calculation, where the redshift of a pair is simply the average  $(z_1 + z_2)/2$ , and the weighting is  $w_1 w_2$  where  $w_i$  is defined above. We determined  $z_{\text{eff}}$  for the bin  $100 < s < 110 h^{-1} \text{ Mpc}$ , although it does not vary significantly with separation. For the combined WiggleZ survey measurement, we found  $z_{\text{eff}} = 0.60$ .

We note that the correlation function measurements are corrected for the effect of redshift blunders in the WiggleZ data catalogue. These are fully quantified in section 3.2 of Blake et al. (2010), and can be well approximated by a scale-independent boost to the correlation function amplitude of  $(1 - f_b)^{-2}$ , where  $f_b \sim 0.05$  is the redshift blunder fraction (which is separately measured for each WiggleZ region).



**Figure 2.** The combined redshift-space correlation function  $\xi(s)$  for WiggleZ survey regions, plotted in the combination  $s^2 \xi(s)$ , where  $s$  is the comoving redshift-space separation. The best-fitting clustering model (varying  $\Omega_m h^2$ ,  $\alpha$  and  $b^2$ ) is overplotted as the solid line. We also show as the dashed line the corresponding ‘no-wiggles’ reference model, constructed from a power spectrum with the same clustering amplitude but lacking BAOs.

### 3.2 Uncertainties: lognormal realizations and covariance matrix

We determined the covariance matrix of the correlation function measurement in each survey region using a large set of lognormal realizations. Jackknife errors, implemented by dividing the survey volume into many subregions, are a poor approximation for the error in the large-scale correlation function because the pair separations of interest are usually comparable to the size of the subregions, which are then not strictly independent. Furthermore, because the WiggleZ data set is not volume-limited and the galaxy number density varies with position, it is impossible to define a set of subregions which are strictly equivalent.

Lognormal realizations are relatively cheap to generate and provide a reasonably accurate galaxy clustering model for the linear and quasi-linear scales which are important for the modelling of baryon oscillations (Coles & Jones 1991). We generated a set of realizations for each survey region using the method described in Blake & Glazebrook (2003) and Glazebrook & Blake (2005). In brief, we started with a model galaxy power spectrum  $P_{\text{mod}}(\mathbf{k})$  consistent with the survey measurement. We then constructed Gaussian realizations of overdensity fields  $\delta_G(\mathbf{r})$  sampled from a second power spectrum  $P_G(\mathbf{k}) \approx P_{\text{mod}}(\mathbf{k})$  (defined below), in which real and imaginary Fourier amplitudes are drawn from a Gaussian distribution with zero mean and standard deviation  $\sqrt{P_G(\mathbf{k})/2}$ . A lognormal overdensity field  $\delta_{\text{LN}}(\mathbf{r}) = \exp(\delta_G) - 1$  is then created, and is used to produce a galaxy density field  $\rho_g(\mathbf{r})$  consistent with the survey window function  $W(\mathbf{r})$ :

$$\rho_g(\mathbf{r}) \propto W(\mathbf{r}) [1 + \delta_{\text{LN}}(\mathbf{r})], \quad (3)$$

where the constant of proportionality is fixed by the size of the final data set. The galaxy catalogue is then Poisson-sampled in cells from the density field  $\rho_g(\mathbf{r})$ . We note that the input power spectrum for the Gaussian overdensity field,  $P_G(\mathbf{k})$ , is constructed to ensure that the final power spectrum of the lognormal overdensity field is consistent with  $P_{\text{mod}}(\mathbf{k})$ . This is achieved using the relation between the correlation functions of Gaussian and lognormal fields,  $\xi_G(\mathbf{r}) = \ln[1 + \xi_{\text{mod}}(\mathbf{r})]$ .

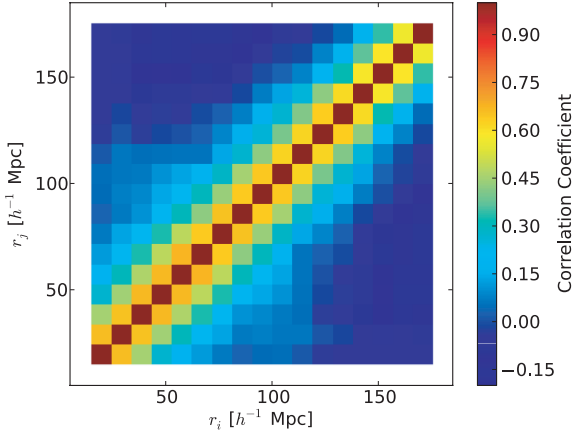
We determined the covariance matrix between bins  $i$  and  $j$  using the correlation function measurements from a large ensemble of lognormal realizations:

$$C_{ij} = \langle \xi_i \xi_j \rangle - \langle \xi_i \rangle \langle \xi_j \rangle, \quad (4)$$

where the angled brackets indicate an average over the realizations. Fig. 3 displays the final covariance matrix resulting from combining the different WiggleZ survey regions in the form of a correlation matrix  $C_{ij}/\sqrt{C_{ii}C_{jj}}$ . The magnitude of the first and second off-diagonal elements of the correlation matrix is typically 0.6 and 0.4, respectively. We find that the jackknife errors on scales of  $100 h^{-1} \text{ Mpc}$  typically exceed the lognormal errors by a factor of  $\approx 50$  per cent, which we can attribute to an overestimation of the number of independent jackknife regions.

### 3.3 Fitting the correlation function : template model and simulations

In this section we discuss the construction of the template fiducial correlation function model  $\xi_{\text{fid,galaxy}}(s)$  which we fitted to the WiggleZ measurement. When fitting the model, we vary a scale distortion parameter  $\alpha$ , a linear normalization factor  $b^2$  and the matter density  $\Omega_m h^2$  which controls both the overall shape of the correlation function and the standard-ruler sound horizon scale. Hence we



**Figure 3.** The amplitude of the cross-correlation  $C_{ij}/\sqrt{C_{ii}C_{jj}}$  of the covariance matrix  $C_{ij}$  for the correlation function measurement plotted in Fig. 2, determined using lognormal realizations.

fitted the model:

$$\xi_{\text{mod}}(s) = b^2 \xi_{\text{fid,galaxy}}(\alpha s). \quad (5)$$

The probability distribution of the scale distortion parameter  $\alpha$ , after marginalizing over  $\Omega_m h^2$  and  $b^2$ , gives the probability distribution of the distance variable  $D_V(z_{\text{eff}}) = \alpha D_{V,\text{fid}}(z_{\text{eff}})$ , where  $z_{\text{eff}} = 0.6$  for our sample (Eisenstein et al. 2005; Padmanabhan & White 2008).  $D_V$ , defined by equation (1), is a composite of the physical angular diameter distance  $D_A(z)$  and Hubble parameter  $H(z)$  which govern tangential and radial galaxy separations, respectively, where  $D_{V,\text{fid}}(z_{\text{eff}}) = 2085.4 \text{ Mpc}$ .

We note that the measured value of  $D_V$  resulting from this fitting process will be independent (to first order) of the fiducial cosmological model adopted for the conversion of galaxy redshifts and angular positions to comoving coordinates. A change in  $D_{V,\text{fid}}$  would result in a shift in the measured position of the acoustic peak. This shift would be compensated for by a corresponding offset in the best-fitting value of  $\alpha$ , leaving the measurement of  $D_V = \alpha D_{V,\text{fid}}$  unchanged (to first order).

An angle-averaged power spectrum  $P(k)$  may be converted into an angle-averaged correlation function  $\xi(s)$  using the spherical Hankel transform:

$$\xi(s) = \frac{1}{2\pi^2} \int dk k^2 P(k) \left[ \frac{\sin(ks)}{ks} \right]. \quad (6)$$

In order to determine the shape of the model power spectrum for a given  $\Omega_m h^2$ , we first generated a linear power spectrum  $P_L(k)$  using the fitting formula of Eisenstein & Hu (1998). This yields a result in good agreement with a CAMB linear power spectrum (Lewis, Challinor & Lasenby 2000), and also produces a wiggle-free reference spectrum  $P_{\text{ref}}(k)$  which possesses the same shape as  $P_L(k)$  but with the baryon oscillation component deleted. This reference spectrum is useful for assessing the statistical significance with which we have detected the acoustic peak. We fixed the values of the other cosmological parameters using our fiducial model:  $h = 0.71$ ,  $\Omega_b h^2 = 0.0226$ ,  $n_s = 0.96$  and  $\sigma_8 = 0.8$ . Our choices for these parameters are consistent with the latest fits to the CMB radiation (Komatsu et al. 2009).

We then corrected the power spectrum for quasi-linear effects. There are two main aspects to the model: a damping of the acoustic peak caused by the displacement of matter due to bulk flows and a distortion in the overall shape of the clustering pattern due to the scale-dependent growth of structure (Eisenstein et al. 2007; Crocce

& Scoccimarro 2008; Matsubara 2008). We constructed our model in a similar manner to Eisenstein et al. (2005). We first incorporated the acoustic peak smoothing by multiplying the power spectrum by a Gaussian damping term  $g(k) = \exp(-k^2 \sigma_v^2)$ :

$$P_{\text{damped}}(k) = g(k) P_L(k) + [1 - g(k)] P_{\text{ref}}(k), \quad (7)$$

where the inclusion of the second term maintains the same small-scale clustering amplitude. The magnitude of the damping can be modelled using perturbation theory (Crocce & Scoccimarro 2008) as

$$\sigma_v^2 = \frac{1}{6\pi^2} \int P_L(k) dk, \quad (8)$$

where  $f = \Omega_m(z)^{0.55}$  is the growth rate of structure. In our fiducial cosmological model,  $\Omega_m h^2 = 0.1361$ , we find  $\sigma_v = 4.5 h^{-1} \text{ Mpc}$ . We checked that this value was consistent with the allowed range when  $\sigma_v$  was varied as a free parameter and fitted to the data.

Next, we incorporated the non-linear boost to the clustering power using the fitting formula of Smith et al. (2003). However, we calculated the non-linear enhancement of power using the input no-wiggles reference spectrum rather than the full linear model including baryon oscillations:

$$P_{\text{damped,NL}}(k) = \left[ \frac{P_{\text{ref,NL}}(k)}{P_{\text{ref}}(k)} \right] \times P_{\text{damped}}(k). \quad (9)$$

Equation (9) is then transformed into a correlation function  $\xi_{\text{damped,NL}}(s)$  using equation (6).

The final component of our model is a scale-dependent galaxy bias term  $B(s)$  relating the galaxy correlation function appearing in equation (5) to the non-linear matter correlation function:

$$\xi_{\text{fid,galaxy}}(s) = B(s) \xi_{\text{damped,NL}}(s), \quad (10)$$

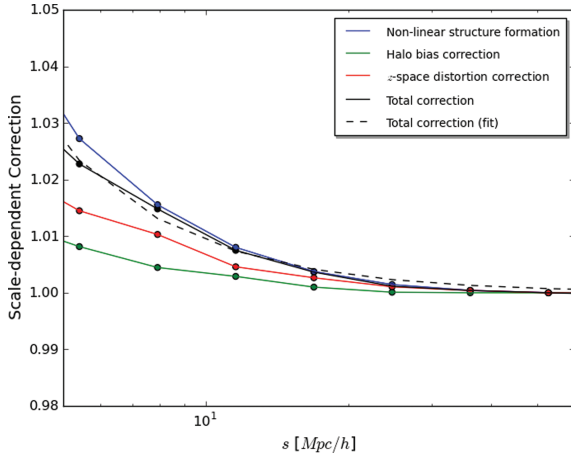
where we note that an overall constant normalization  $b^2$  has already been separated in equation (5) so that  $B(s) \rightarrow 1$  at large  $s$ .

We determined the form of  $B(s)$  using halo catalogues extracted from the GigggleZ dark matter simulation. This  $N$ -body simulation has been generated specifically in support of WiggleZ survey science, and consists of  $2160^3$  particles evolved in a  $1 h^{-3} \text{ Gpc}^3$  box using a *Wilkinson Microwave Anisotropy Probe 5* (WMAP5) cosmology (Komatsu et al. 2009). We deduced  $B(s)$  using the non-linear redshift-space halo correlation functions and non-linear dark matter correlation function of the simulation. We found that a satisfactory fitting formula for the scale-dependent bias over the scales of interest is

$$B(s) = 1 + (s/s_0)^\gamma. \quad (11)$$

We performed this procedure for several contiguous subsets of 250 000 haloes rank-ordered by their maximum circular velocity (a robust proxy for halo mass). The best-fitting parameters of equation (11) for the subset which best matches the large-scale WiggleZ clustering amplitude are  $s_0 = 0.32 h^{-1} \text{ Mpc}$  and  $\gamma = -1.36$ . Our measurement of the scale-dependent bias correction using the real-space and redshift-space correlation functions from the GigggleZ simulation is plotted in Fig. 4. We note that the magnitude of the scale-dependent correction from this term is  $\sim 1$  per cent for a scale  $s \sim 10 h^{-1} \text{ Mpc}$ , which is far smaller than the  $\sim 10$  per cent magnitude of such effects for more strongly biased galaxy samples such as LRGs (Eisenstein et al. 2005). This greatly reduces the potential for systematic error due to a failure to model correctly scale-dependent galaxy bias effects.



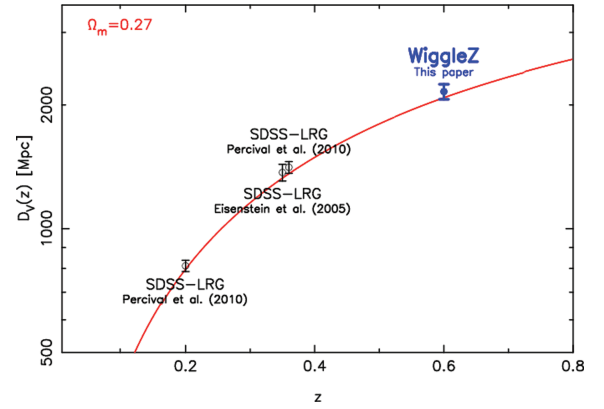


**Figure 4.** The scale-dependent correction to the non-linear real-space dark matter correlation function for haloes with maximum circular velocity  $V_{\text{max}} \approx 125 \text{ km s}^{-1}$ , which possess the same amplitude of large-scale clustering as WiggleZ galaxies. The green line is the ratio of the real-space halo correlation function to the real-space non-linear dark matter correlation function. The red line is the ratio of the redshift-space halo correlation function to the real-space halo correlation function. The black line, the product of the red and green lines, is the scale-dependent bias correction  $B(s)$  which we fitted with the model of equation (11), shown as the dashed black line. The blue line is the ratio of the real-space non-linear to linear correlation function.

### 3.4 Extraction of $D_V$

We fitted the galaxy correlation function template model described above to the WiggleZ survey measurement, varying the matter density  $\Omega_m h^2$ , the scale distortion parameter  $\alpha$  and the galaxy bias  $b^2$ . Our default fitting range was  $10 < s < 180 h^{-1} \text{ Mpc}$  (following Eisenstein et al. 2005), where  $10 h^{-1} \text{ Mpc}$  is an estimate of the minimum scale of validity for the quasi-linear theory described in Section 3.3. In the following, we assess the sensitivity of the parameter constraints to the fitting range.

We minimized the  $\chi^2$  statistic using the full data covariance matrix, assuming that the probability of a model was proportional to  $\exp(-\chi^2/2)$ . The best-fitting parameters were  $\Omega_m h^2 = 0.132 \pm$



**Figure 5.** Measurements of the distance–redshift relation using the BAO standard ruler from LRG samples (Eisenstein et al. 2005; Percival et al. 2010) and the current WiggleZ analysis. The results are compared to a fiducial flat  $\Lambda$ CDM cosmological model with matter density  $\Omega_m = 0.27$ .

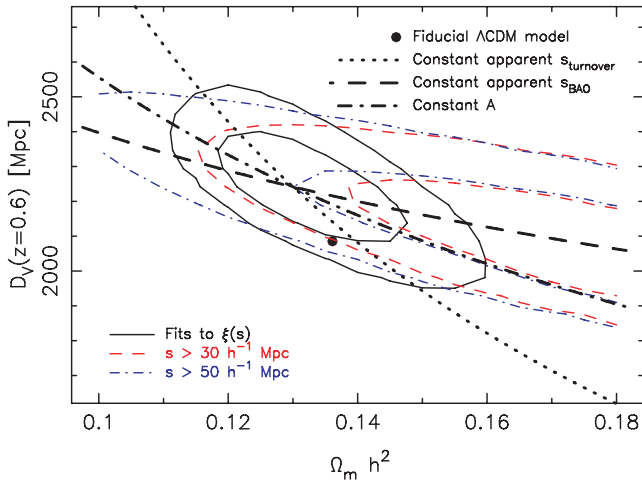
$0.011$ ,  $\alpha = 1.075 \pm 0.055$  and  $b^2 = 1.21 \pm 0.11$ , where the errors in each parameter are produced by marginalizing over the remaining two parameters. The minimum value of  $\chi^2$  is 14.9 for 14 degrees of freedom (17 bins minus three fitted parameters), indicating an acceptable fit to the data. In Fig. 2 we compare the best-fitting correlation function model to the WiggleZ data points. The results of the parameter fits are summarized for ease of reference in Table 1.

Our measurement of the scale distortion parameter  $\alpha$  may be translated into a constraint on the distance scale  $D_V = \alpha D_{V,\text{fid}} = 2234.9 \pm 115.2 \text{ Mpc}$ , corresponding to a 5.2 per cent measurement of the distance scale at  $z = 0.60$ . This accuracy is comparable to that reported by Eisenstein et al. (2005) for the analysis of the SDSS DR3 LRG sample at  $z = 0.35$ . Fig. 5 compares our measurement of the distance–redshift relation with those from the LRG samples analysed by Eisenstein et al. (2005) and Percival et al. (2010).

The 2D probability contours for the parameters  $\Omega_m h^2$  and  $D_V(z = 0.6)$ , marginalizing over  $b^2$ , are displayed in Fig. 6. Following Eisenstein et al. (2005) we indicate three degeneracy directions in this parameter space. The first direction (the dashed line in Fig. 6) corresponds to a constant measured acoustic peak separation, i.e.  $r_s(z_d)/D_V(z = 0.6) = \text{constant}$ . We used the fitting

**Table 1.** Results of fitting a three-parameter model ( $\Omega_m h^2$ ,  $\alpha$ ,  $b^2$ ) to WiggleZ measurements of four different clustering statistics for various ranges of scales. The top four entries, in the upper part, correspond to our fiducial choices of fitting range for each statistic. The fitted scales  $\alpha$  are converted into measurements of  $D_V$  and two BAO distilled parameters,  $A$  and  $r_s(z_d)/D_V$ , which are introduced in Section 7. The final column lists the measured value of  $D_V$  when the parameter  $\Omega_m h^2$  is left fixed at its fiducial value and only the bias  $b^2$  is marginalized. We recommend using  $A(z = 0.6)$  as measured by the correlation function  $\xi(s)$  for the scale range  $10 < s < 180 h^{-1} \text{ Mpc}$ , highlighted in bold, as the most appropriate WiggleZ measurement for deriving BAO constraints on cosmological parameters.

Statistic	Scale range	$\Omega_m h^2$	$D_V(z = 0.6)$ (Mpc)	$A(z = 0.6)$	$r_s(z_d)/D_V(z = 0.6)$	$D_V(z = 0.6)$ fixing $\Omega_m h^2$
$\xi(s)$	$10 < s < 180 h^{-1} \text{ Mpc}$	$0.132 \pm 0.011$	$2234.9 \pm 115.2$	<b><math>0.452 \pm 0.018</math></b>	$0.0692 \pm 0.0033$	$2216.5 \pm 78.9$
$P(k)$ [full]	$0.02 < k < 0.2 h \text{ Mpc}^{-1}$	$0.134 \pm 0.008$	$2160.7 \pm 132.3$	$0.440 \pm 0.020$	$0.0711 \pm 0.0038$	$2141.0 \pm 97.5$
$P(k)$ [wiggles]	$0.02 < k < 0.2 h \text{ Mpc}^{-1}$	$0.163 \pm 0.017$	$2135.4 \pm 156.7$	$0.461 \pm 0.030$	$0.0699 \pm 0.0045$	$2197.2 \pm 119.1$
$w_0(r)$	$10 < r < 180 h^{-1} \text{ Mpc}$	$0.130 \pm 0.011$	$2279.2 \pm 142.4$	$0.456 \pm 0.021$	$0.0680 \pm 0.0037$	$2238.2 \pm 104.6$
$\xi(s)$	$30 < s < 180 h^{-1} \text{ Mpc}$	$0.166 \pm 0.014$	$2127.7 \pm 127.9$	$0.475 \pm 0.025$	$0.0689 \pm 0.0031$	$2246.8 \pm 102.6$
$\xi(s)$	$50 < s < 180 h^{-1} \text{ Mpc}$	$0.164 \pm 0.016$	$2129.2 \pm 140.8$	$0.474 \pm 0.025$	$0.0690 \pm 0.0031$	$2240.1 \pm 104.7$
$P(k)$ [full]	$0.02 < k < 0.1 h \text{ Mpc}^{-1}$	$0.150 \pm 0.020$	$2044.7 \pm 253.0$	$0.441 \pm 0.034$	$0.0733 \pm 0.0073$	$2218.1 \pm 128.4$
$P(k)$ [full]	$0.02 < k < 0.3 h \text{ Mpc}^{-1}$	$0.137 \pm 0.007$	$2132.1 \pm 109.2$	$0.441 \pm 0.017$	$0.0716 \pm 0.0033$	$2148.9 \pm 79.9$
$P(k)$ [wiggles]	$0.02 < k < 0.1 h \text{ Mpc}^{-1}$	$0.160 \pm 0.020$	$2240.7 \pm 235.8$	$0.466 \pm 0.034$	$0.0678 \pm 0.0070$	$2277.9 \pm 187.5$
$P(k)$ [wiggles]	$0.02 < k < 0.3 h \text{ Mpc}^{-1}$	$0.161 \pm 0.019$	$2114.5 \pm 132.4$	$0.455 \pm 0.026$	$0.0706 \pm 0.0037$	$2171.4 \pm 98.0$
$w_0(r)$	$30 < r < 180 h^{-1} \text{ Mpc}$	$0.127 \pm 0.018$	$2288.8 \pm 157.3$	$0.455 \pm 0.027$	$0.0681 \pm 0.0037$	$2251.6 \pm 111.7$
$w_0(r)$	$50 < r < 180 h^{-1} \text{ Mpc}$	$0.164 \pm 0.016$	$2190.0 \pm 146.2$	$0.466 \pm 0.023$	$0.0673 \pm 0.0036$	$2282.1 \pm 109.8$



**Figure 6.** Probability contours of the physical matter density  $\Omega_m h^2$  and distance scale  $D_V(z = 0.6)$  obtained by fitting to the WiggleZ survey combined correlation function  $\xi(s)$ . Results are compared for different ranges of fitted scales  $s_{\min} < s < 180 h^{-1}$  Mpc. The (black solid, red dashed, blue dot-dashed) contours correspond to fitting for  $s_{\min} = (10, 30, 50) h^{-1}$  Mpc, respectively. The heavy dashed and dotted lines are the degeneracy directions which are expected to result from fits involving respectively just the acoustic oscillations and just the shape of a pure CDM power spectrum. The heavy dot-dashed line represents a constant value of the acoustic ‘A’ parameter introduced by Eisenstein et al. (2005), which is the parameter best-measured by our correlation function data. The solid circle represents the location of our fiducial cosmological model. The two contour levels in each case enclose regions containing 68 and 95 per cent of the likelihood.

formula quoted in Percival et al. (2010) to determine  $r_s(z_d)$  as a function of  $\Omega_m h^2$  (given our fiducial value of  $\Omega_b h^2 = 0.0226$ ); we find that  $r_s(z_d) = 152.6$  Mpc for our fiducial cosmological model. The second degeneracy direction (the dotted line in Fig. 6) corresponds to a constant measured shape of a CDM power spectrum, i.e.  $D_V(z = 0.6) \times \Omega_m h^2 = \text{constant}$ . In such models, the matter transfer function at recombination can be expressed as a function of  $q = k/\Omega_m h^2$  (Bardeen et al. 1986). Given that changing  $D_V$  corresponds to a scaling of  $k \propto D_{V,\text{fid}}/D_V$ , we recover that the measured power spectrum shape depends on  $D_V \Omega_m h^2$ . The principle degeneracy axis of our measurement lies between these two curves, suggesting that both the correlation function shape and acoustic peak information are driving our measurement of  $D_V$ . The third degeneracy direction we plot (the dot-dashed line in Fig. 6), which matches our measurement, corresponds to a constant value of the acoustic parameter  $A(z) \equiv D_V(z) \sqrt{\Omega_m H_0^2}/cz$  introduced by Eisenstein et al. (2005). We present our fits for this parameter in Section 7.

In Fig. 6 we also show probability contours resulting from fits to a restricted range of separations  $s > 30$  and  $50 h^{-1}$  Mpc. In both cases, the contours become significantly more extended and the long axis shifts into alignment with the case of the acoustic peak alone driving the fits. The restricted fitting range no longer enables us to perform an accurate determination of the value of  $\Omega_m h^2$  from the shape of the clustering pattern alone.

### 3.5 Significance of the acoustic peak detection

In order to assess the importance of the baryon acoustic peak in constraining this model, we repeated the parameter fit replacing the model correlation function with one generated using a ‘no-wiggles’ reference power spectrum  $P_{\text{ref}}(k)$ , which possesses the same amplitude and overall shape as the original matter power

spectrum but lacks the baryon oscillation features [i.e. we replaced  $P_L(k)$  with  $P_{\text{ref}}(k)$  in equation 7]. The minimum value obtained for the  $\chi^2$  statistic in this case was 25.0, indicating that the model containing baryon oscillations was favoured by  $\Delta\chi^2 = 10.1$ . This corresponds to a detection of the acoustic peak with a statistical significance of  $3.2\sigma$ . Furthermore, the value and error obtained for the scale distortion parameter in the no-wiggles model was  $\alpha = 0.80 \pm 0.17$ , representing a degradation of the error in  $\alpha$  by a factor of 3. This also suggests that the acoustic peak is important for establishing the distance constraints from our measurement.

As an alternative approach for assessing the significance of the acoustic peak, we changed the fiducial baryon density to  $\Omega_b = 0$  and repeated the parameter fit. The minimum value obtained for the  $\chi^2$  statistic was now 22.7 and the value and marginalized error determined for the scale distortion parameter was  $\alpha = 0.80 \pm 0.12$ , reaffirming the significance of our detection of the baryon wiggles.

If we restrict the correlation function fits to the range  $50 < s < 130 h^{-1}$  Mpc, further reducing the influence of the overall shape of the clustering pattern on the goodness of fit, we find that our fiducial model has a minimum  $\chi^2 = 5.9$  (for 5 degrees of freedom) and the ‘no-wiggles’ reference spectrum produces a minimum  $\chi^2 = 13.1$ . Even for this restricted range of scales, the model containing baryon oscillations was therefore favoured by  $\Delta\chi^2 = 7.2$ .

### 3.6 Sensitivity to the clustering model

In this section we investigate the systematic dependence of our measurement of  $D_V(z = 0.6)$  on the model used to describe the quasi-linear correlation function. We considered five modelling approaches proposed in the literature.

(i) *Model 1.* Our fiducial model described in Section 3.3 following Eisenstein et al. (2005), in which the quasi-linear damping of the acoustic peak was modelled by an exponential factor  $g(k) = \exp(-k^2 \sigma_v^2)$ ,  $\sigma_v$  is determined from linear theory via equation (8), and the small-scale power was restored by adding a term  $[1 - g(k)]$  multiplied by the wiggle-free reference spectrum (equation 7).

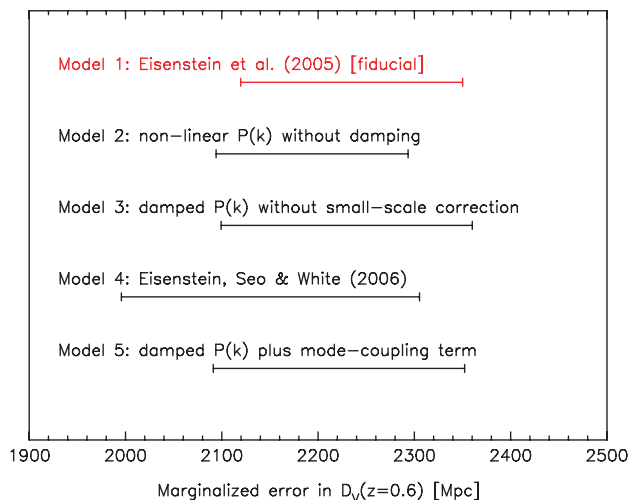
(ii) *Model 2.* No quasi-linear damping of the acoustic peak was applied, i.e.  $\sigma_v = 0$ .

(iii) *Model 3.* The term restoring the small-scale power,  $[1 - g(k)]P_{\text{ref}}(k)$  in equation (7), was omitted.

(iv) *Model 4.*  $P_{\text{damped}}(k)$  in equation (7) was generated using equation (14) of Eisenstein, Seo & White (2007), which implements different damping coefficients in the tangential and radial directions.

(v) *Model 5.* The quasi-linear matter correlation function was generated using equation (10) of Sanchez et al. (2009), following Crocce & Scoccimarro (2008), which includes the additional contribution of a ‘mode-coupling’ term. We set the coefficient  $A_{\text{MC}} = 1$  in this equation (rather than introduce an additional free parameter).

Fig. 7 compares the measurements of  $D_V(z = 0.6)$  from the correlation function data, marginalized over  $\Omega_m h^2$  and  $b^2$ , assuming each of these models. The agreement amongst the best-fitting measurements is excellent, and the minimum  $\chi^2$  statistics imply a good fit to the data in each case. We conclude that systematic errors associated with modelling the correlation function are not significantly affecting our results. The error in the distance measurement is determined by the amount of damping of the acoustic peak, which controls the precision with which the standard ruler may be applied. The lowest distance error is produced by Model 2 which neglects damping; the greatest distance error is associated with Model 4, in which the damping is enhanced along the line of sight (see equation 13 in Eisenstein et al. 2007).



**Figure 7.** Measurements of  $D_V(z = 0.6)$  from the galaxy correlation function, marginalizing over  $\Omega_m h^2$  and  $b^2$ , comparing five different models for the quasi-linear correlation function as detailed in the text. The measurements are consistent, suggesting that systematic modelling errors are not significantly affecting our results.

## 4 POWER SPECTRUM

### 4.1 Measurements and covariance matrix

The power spectrum is a second commonly used method for quantifying the galaxy clustering pattern, which is complementary to the correlation function. It is calculated using a Fourier decomposition of the density field in which (contrary to the correlation function) the maximal signal-to-noise ratio is achieved on large, linear or quasi-linear scales (at low wavenumbers) and the measurement of small-scale power (at high wavenumbers) is limited by shot noise. However, also in contrast to the correlation function, small-scale effects such as shot noise influence the measured power at all wavenumbers, and the baryon oscillation signature appears as a series of decaying harmonic peaks and troughs at different wavenumbers. In aesthetic terms this diffusion of the baryon oscillation signal is disadvantageous.

We estimated the galaxy power spectrum for each separate WiggleZ survey region using the direct Fourier methods introduced by Feldman, Kaiser & Peacock (1994, hereafter FKP). Our methodology is fully described in section 3.1 of Blake et al. (2010); we give a brief summary here. First, we map the angle-redshift survey cone into a cuboid of comoving coordinates using a fiducial flat  $\Lambda$ CDM cosmological model with matter density  $\Omega_m = 0.27$ . We gridded the catalogue in cells using nearest grid point assignment ensuring that the Nyquist frequencies in each direction were much higher than the Fourier wavenumbers of interest (we corrected the power spectrum measurement for the small bias introduced by this gridding). We then applied a Fast Fourier transform to the grid, optimally weighting each pixel by  $1/(1 + nP_0)$ , where  $n$  is the galaxy number density in the pixel (determined using the selection function) and  $P_0 = 5000 h^{-3} \text{Mpc}^3$  is a characteristic power spectrum amplitude. The Fast Fourier transform of the selection function is then used to construct the final power spectrum estimator using equation 13 in Blake et al. (2010). The measurement is corrected for the effect of redshift blunders using Monte Carlo survey simulations as described in section 3.2 of Blake et al. (2010). We measured each power spectrum in wavenumber bins of width  $0.01 h \text{Mpc}^{-1}$  between  $k = 0$  and  $0.3 h \text{Mpc}^{-1}$ , and determined the covariance ma-

trix of the measurement in these bins by implementing the sums in Fourier space described by FKP (see Blake et al. 2010, equations 20–22). The FKP errors agree with those obtained from lognormal realizations within 10 per cent at all scales.

In order to detect and fit for the baryon oscillation signature in the WiggleZ galaxy power spectrum, we need to stack together the measurements in the individual survey regions and redshift slices. This requires care because each subregion possesses a different selection function, and therefore each power spectrum measurement corresponds to a different convolution of the underlying power spectrum model. Furthermore, the non-linear component of the underlying model varies with redshift, due to non-linear evolution of the density and velocity power spectra. Hence the observed power spectrum in general has a systematically different slope in each subregion, which implies that the baryon oscillation peaks lie at slightly different wavenumbers. If we stacked together the raw measurements, there would be a significant washing-out of the acoustic peak structure.

Therefore, before combining the measurements, we made a correction to the shape of the various power spectra to bring them into alignment. We wish to avoid spuriously enhancing the oscillatory features when making this correction. Our starting point is therefore a fiducial power spectrum model generated from the Eisenstein & Hu (1998) ‘no-wiggles’ reference linear power spectrum, which defines the fiducial slope to which we correct each measurement. First, we modified this reference function into a redshift-space non-linear power spectrum, using an empirical redshift-space distortion model fitted to the 2D power spectrum split into tangential and radial bins (see Blake et al. 2011a). The redshift-space distortion is modelled by a coherent-flow parameter  $\beta$  and a pairwise velocity dispersion parameter  $\sigma_v$ , which were fitted independently in each of the redshift slices. We convolved this redshift-space non-linear reference power spectrum with the selection function in each subregion, and our correction factor for the measured power spectrum is then the ratio of this convolved function to the original real-space linear reference power spectrum. After applying this correction to the data and covariance matrix we combined the resulting power spectra using inverse-variance weighting.

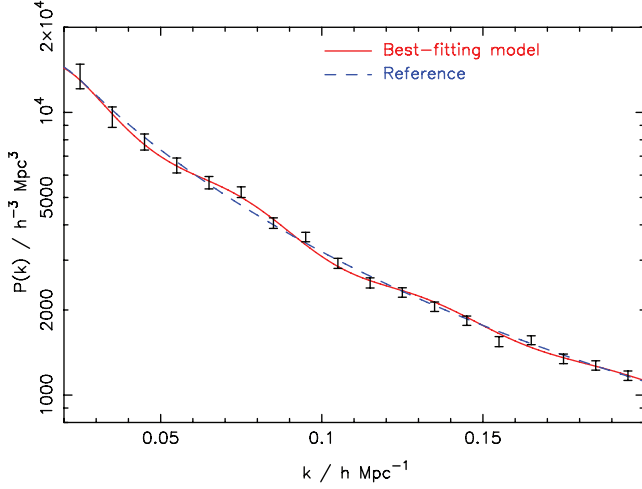
Figs 8 and 9, respectively, display the combined power spectrum data and that data divided through by the combined no-wiggles reference spectrum in order to reveal any signature of acoustic oscillations more clearly. We note that there is a significant enhancement of power at the position of the first harmonic,  $k \approx 0.075 h \text{Mpc}^{-1}$ . The other harmonics are not clearly detected with the current data set, although the model is nevertheless a good statistical fit. Fig. 10 displays the final power spectrum covariance matrix, resulting from combining the different WiggleZ survey regions, in the form of a correlation matrix  $C_{ij}/\sqrt{C_{ii}C_{jj}}$ . We note that there is very little correlation between separate  $0.01 h \text{Mpc}^{-1}$  power spectrum bins.

We note that our method for combining power spectrum measurements in different subregions only corrects for the convolution effect of the window function on the overall power spectrum shape, and does not undo the smoothing of the BAO signature in each window. We therefore expect the resulting BAO detection in the combined power spectrum may have somewhat lower significance than that in the combined correlation function.

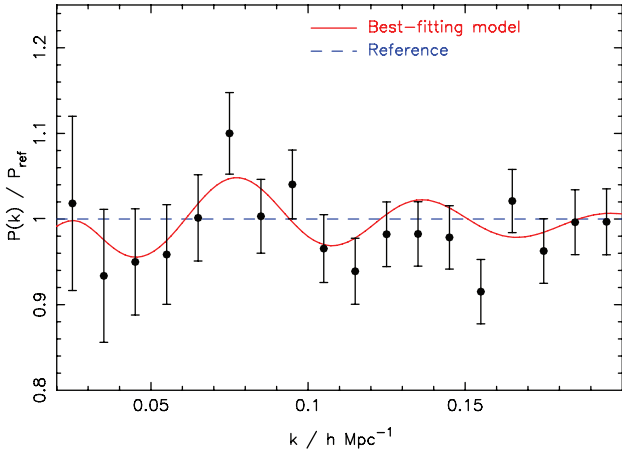
### 4.2 Extraction of $D_V$

We investigated two separate methods for fitting the scale distortion parameter to the power spectrum data. Our first approach used the whole shape of the power spectrum including any baryonic





**Figure 8.** The power spectrum obtained by stacking measurements in different WiggleZ survey regions using the method described in Section 4.1. The best-fitting power spectrum model (varying  $\Omega_m h^2$ ,  $\alpha$  and  $b^2$ ) is overplotted as the solid line. We also show the corresponding ‘no-wiggles’ reference model as the dashed line, constructed from a power spectrum with the same clustering amplitude but lacking BAOs.



**Figure 9.** The combined WiggleZ survey power spectrum of Fig. 8 divided by the smooth reference spectrum to reveal the signature of baryon oscillations more clearly. We detect the first harmonic peak in Fourier space.

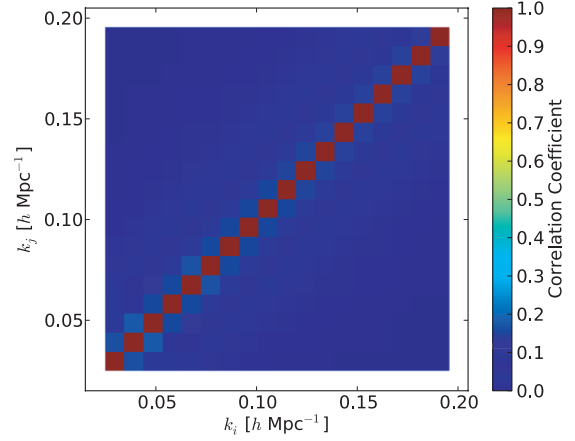
signature. We generated a template model non-linear power spectrum  $P_{\text{fid}}(k)$  parametrized by  $\Omega_m h^2$ , which we took as equation (9) in Section 3.3, and fitted the model:

$$P_{\text{mod}}(k) = b^2 P_{\text{fid}}(k/\alpha), \quad (12)$$

where  $\alpha$  now appears in the denominator (as opposed to the numerator of equation 5) due to the switch from real space to Fourier space. As in the case of the correlation function, the probability distribution of  $\alpha$ , after marginalizing over  $\Omega_m h^2$  and  $b^2$ , can be connected to the measurement of  $D_V(z_{\text{eff}})$ . We determined the effective redshift of the power spectrum estimate by weighting each pixel in the selection function by its contribution to the power spectrum error:

$$z_{\text{eff}} = \sum_x z \left[ \frac{n_g(\mathbf{x}) P_g}{1 + n_g(\mathbf{x}) P_g} \right]^2, \quad (13)$$

where  $n_g(\mathbf{x})$  is the galaxy number density in each grid cell  $\mathbf{x}$  and  $P_g$  is the characteristic galaxy power spectrum amplitude, which we evaluated at a scale  $k = 0.1 h \text{ Mpc}^{-1}$ . We obtained an effective



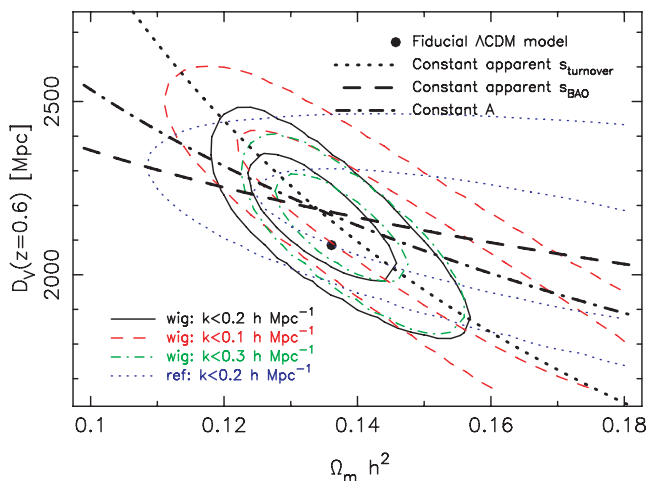
**Figure 10.** The amplitude of the cross-correlation  $C_{ij}/\sqrt{C_{ii}C_{jj}}$  of the covariance matrix  $C_{ij}$  for the power spectrum measurement, determined using the FKP estimator. The amplitude of the off-diagonal elements of the covariance matrix is very low.

redshift  $z_{\text{eff}} = 0.583$ . In order to enable comparison with the correlation function fits, we applied the best-fitting value of  $\alpha$  at  $z = 0.6$ .

Our second approach to fitting the power spectrum measurement used only the information contained in the baryon oscillations. We divided the combined WiggleZ power spectrum data by the corresponding combined no-wiggles reference spectrum, and when fitting models we divided each trial power spectrum by its corresponding reference spectrum prior to evaluating the  $\chi^2$  statistic.

We restricted our fits to Fourier wavenumbers  $0.02 < k < 0.2 h \text{ Mpc}^{-1}$ , where the upper limit is an estimate of the range of reliability of the quasi-linear power spectrum modelling. We investigate below the sensitivity of the best-fitting parameters to the fitting range. For the first method, fitting to the full power spectrum shape, the best-fitting parameters and 68 per cent confidence ranges were  $\Omega_m h^2 = 0.134 \pm 0.008$  and  $\alpha = 1.050 \pm 0.064$ , where the errors in each parameter are produced by marginalizing over the remaining two parameters. The minimum value of  $\chi^2$  was 12.4 for 15 degrees of freedom (18 bins minus three fitted parameters), indicating an acceptable fit to the data. We can convert the constraint on the scale distortion parameter into a measured distance  $D_V(z = 0.6) = 2160.7 \pm 132.3 \text{ Mpc}$ . The 2D probability distribution of  $\Omega_m h^2$  and  $D_V(z = 0.6)$ , marginalizing over  $b^2$ , is displayed as the solid contours in Fig. 11. In this figure we reproduce the same degeneracy lines discussed in Section 3.4, which are expected to result from fits involving just the acoustic oscillations and just the shape of a pure CDM power spectrum. We note that the long axis of our probability contours is oriented close to the latter line, indicating that the acoustic peak is not exerting a strong influence on fits to the full WiggleZ power spectrum shape. Comparison of Fig. 11 with Fig. 6 shows that fits to the WiggleZ galaxy correlation function are currently more influenced by the BAOs than the power spectrum. This is attributable to the signal being stacked at a single scale in the correlation function, in this case of a moderate BAO detection.

For the second method, fitting to just the baryon oscillations, the best-fitting parameters and 68 per cent confidence ranges were  $\Omega_m h^2 = 0.163 \pm 0.017$  and  $\alpha = 1.000 \pm 0.073$ . Inspection of the 2D probability contours of  $\Omega_m h^2$  and  $\alpha$ , which are shown as the dotted contours in Fig. 11, indicates that a significant degeneracy has opened up parallel to the line of constant apparent BAO scale (as expected). Increasing  $\Omega_m h^2$  decreases the standard-ruler scale,



**Figure 11.** Probability contours of the physical matter density  $\Omega_m h^2$  and distance scale  $D_V(z=0.6)$  obtained by fitting to the WiggleZ survey combined power spectrum. Results are compared for different ranges of fitted scales  $0.02 < k < k_{\max}$  and methods. The (red dashed, black solid, green dot-dashed) contours correspond to fits of the full power spectrum model for  $k_{\max} = (0.1, 0.2, 0.3) h \text{ Mpc}^{-1}$ , respectively. The blue dotted contours result from fitting to the power spectrum divided by a smooth no-wiggles reference spectrum (with  $k_{\max} = 0.2 h \text{ Mpc}^{-1}$ ). Degeneracy directions and likelihood contour levels are plotted as in Fig. 6.

but the positions of the acoustic peaks may be brought back into line with the data by applying a lower scale distortion parameter  $\alpha$ . Low values of  $\Omega_m h^2$  are ruled out because the resulting amplitude of baryon oscillations is too high (given that  $\Omega_b h^2$  is fixed). We also plot in Fig. 11 the probability contours resulting from fitting different ranges of Fourier scales  $k < 0.1 h \text{ Mpc}^{-1}$  and  $k < 0.3 h \text{ Mpc}^{-1}$ . The 68 per cent confidence regions generated for these different cases overlap.

We assessed the significance with which acoustic features are detected in the power spectrum using a method similar to our treatment of the correlation function in Section 3. We repeated the parameter fit for  $(\Omega_m h^2, \alpha, b^2)$  using the ‘no-wiggles’ reference power spectrum in place of the full model power spectrum. The minimum value obtained for the  $\chi^2$  statistic in this case was 15.8, indicating that the model containing baryon oscillations was favoured by only  $\Delta\chi^2 = 3.3$ . This is consistent with the direction of the long axis of the probability contours in Fig. 11, which suggests that the baryon oscillations are not driving the fits to the full power spectrum shape.

## 5 BAND-FILTERED CORRELATION FUNCTION

### 5.1 Measurements and covariance matrix

Xu et al. (2010) introduced a new statistic for the measurement of the acoustic peak in configuration space, which they describe as an advantageous approach for band filtering the information. They proposed estimating the quantity:

$$w_0(r) = 4\pi \int_0^r \frac{ds}{r} \left(\frac{s}{r}\right)^2 \xi(s) W\left(\frac{s}{r}\right), \quad (14)$$

where  $\xi(s)$  is the two-point correlation function as a function of separation  $s$  and

$$W(x) = (2x)^2(1-x)^2 \left(\frac{1}{2} - x\right), \quad 0 < x < 1 \quad (15)$$

$$= 0, \quad \text{otherwise}, \quad (16)$$

in terms of  $x = (s/r)^3$ . This filter localizes the acoustic information in a single feature at the acoustic scale in a similar manner to the correlation function, which is beneficial for securing a robust, model-independent detection. However, the form of the filter function  $W(x)$  advantageously reduces the sensitivity to small-scale power (which is difficult to model due to non-linear effects) and large-scale power (which is difficult to measure because it is subject to uncertainties regarding the mean density of the sample), combining the respective advantages of the correlation function and power spectrum approaches.

Xu et al. (2010) proposed that  $w_0(r)$  should be estimated as a weighted sum over galaxy pairs  $i$

$$w_0(r) = \text{DD}_{\text{filtered}}(r) = \frac{2}{N_D n_D} \sum_{i=1}^{N_{\text{pairs}}} \frac{W(s_i/r)}{\phi(s_i, \mu_i)}, \quad (17)$$

where  $s_i$  is the separation of pair  $i$ ,  $\mu_i$  is the cosine of the angle of the separation vector to the line of sight,  $N_D$  is the number of data galaxies, and  $n_D$  is the average galaxy density which we simply define as  $N_D/V$  where  $V$  is the volume of a cuboid enclosing the survey cone. The function  $\phi(s, \mu)$  describes the edge effects due to the survey boundaries and is normalized so that the number of random pairs in a bin  $(s \rightarrow s + ds, \mu \rightarrow \mu + d\mu)$  is

$$\text{RR}(s, \mu) = 2\pi n_D N_D s^2 \phi(s, \mu) ds d\mu, \quad (18)$$

where  $\phi = 1$  for a uniform, infinite survey. We determined the function  $\phi(s, \mu)$  used in equation (17) by binning the pair counts  $\text{RR}(s, \mu)$  for many random sets in fine bins of  $s$  and  $\mu$ . We then fitted a parametrized model

$$\phi(s, \mu) = \sum_{n=0}^3 a_n(s) \mu^{2n} \quad (19)$$

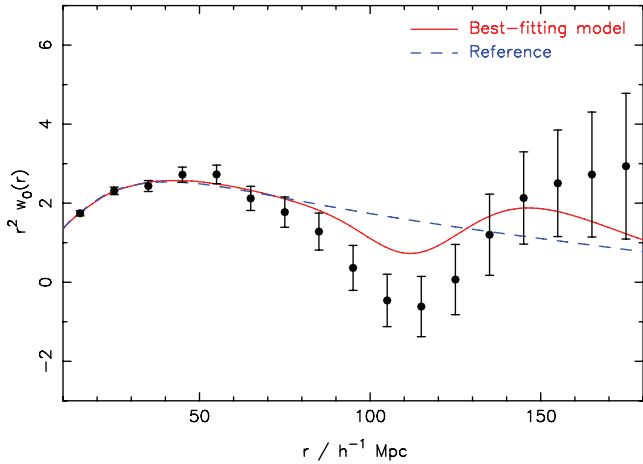
to the result in bins of  $s$ , and used the coefficients  $a_n(s)$  to generate the value of  $\phi$  for each galaxy pair.

We note that our equation (17) contains an extra factor of 2 compared to equation (12) of Xu et al. (2010) because we define the quantity as a sum over unique pairs, rather than all pairs. We also propose to modify the estimator to introduce a ‘DR’ term by analogy with the correlation function estimator of equation (2), in order to correct for the distribution of data galaxies with respect to the boundaries of the sample:

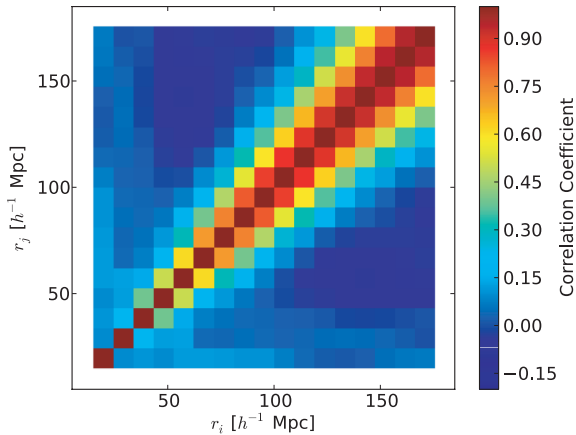
$$w_0(r) = \text{DD}_{\text{filtered}}(r) - \text{DR}_{\text{filtered}}(r), \quad (20)$$

where  $\text{DR}_{\text{filtered}}(r)$  is estimated using equation (17), but summing over data-random pairs and excluding the initial factor of 2. We used our lognormal realizations to determine that this modified estimator of equation (20) produces a result with lower bias and variance compared to equation (17). We used equation (20) to measure the band-filtered correlation function of each WiggleZ region for 17 values of  $r$  spaced by  $10 h^{-1} \text{ Mpc}$  between  $15$  and  $175 h^{-1} \text{ Mpc}$ .

We determined the covariance matrix  $C_{ij}$  of our estimator using the ensemble of lognormal realizations for each survey region. We note that for our data set the amplitude of the diagonal errors  $\sqrt{C_{ii}}$  determined by lognormal realizations is typically  $\sim 5$  times greater than jackknife errors and  $\sim 3$  times higher than obtained by evaluating equation (13) in Xu et al. (2010) which estimates the covariance



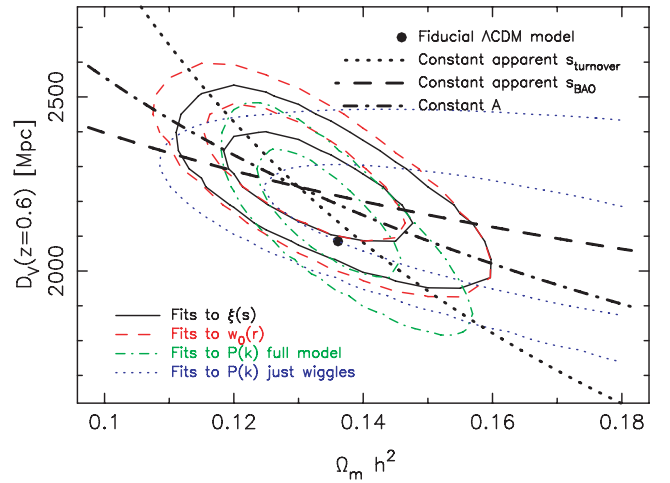
**Figure 12.** The band-filtered correlation function  $w_0(r)$  for the combined WiggleZ survey regions, plotted in the combination  $r^2 w_0(r)$ . The best-fitting clustering model (varying  $\Omega_m h^2$ ,  $\alpha$  and  $b^2$ ) is overplotted as the solid line. We also show the corresponding ‘no-wiggles’ reference model, constructed from a power spectrum with the same clustering amplitude but lacking BAOs. We note that the high covariance of the data points for this estimator implies that (despite appearances) the solid line is a good statistical fit to the data.



**Figure 13.** The amplitude of the cross-correlation  $C_{ij}/\sqrt{C_{ii}C_{jj}}$  of the covariance matrix  $C_{ij}$  for the band-filtered correlation function measurement plotted in Fig. 12, determined using lognormal realizations.

matrix in the Gaussian limit. Given the likely drawbacks of jackknife errors (the lack of independence of the jackknife regions on large scales) and Gaussian errors (which fail to incorporate the survey selection function), the lognormal errors should provide by far the best estimate of the covariance matrix for this measurement.

We constructed the final measurement of the band-filtered correlation function by stacking the individual measurements in different survey regions with inverse-variance weighting. Fig. 12 displays our measurement. We detect clear evidence of the expected dip in  $w_0(r)$  at the acoustic scale. Fig. 13 displays the final covariance matrix of the band-filtered correlation function resulting from combining the different WiggleZ survey regions in the form of a correlation matrix  $C_{ij}/\sqrt{C_{ii}C_{jj}}$ . We note that the nature of the  $w_0(r)$  estimator, which depends on the correlation function at all scales  $s < r$ , implies that the data points in different bins of  $r$  are highly correlated, and the correlation coefficient increases with  $r$ . At the acoustic scale, neighbouring  $10 h^{-1}$  Mpc bins are correlated at the  $\sim 85$  per cent



**Figure 14.** A comparison of the probability contours of  $\Omega_m h^2$  and  $D_V(z=0.6)$  resulting from fitting different clustering statistics measured from the WiggleZ survey: the correlation function for scale range  $10 < s < 180 h^{-1}$  Mpc (the solid black contours); the band-filtered correlation function for scale range  $10 < r < 180 h^{-1}$  Mpc (the dashed red contours); the full power spectrum shape for scale range  $0.02 < k < 0.2 h \text{ Mpc}^{-1}$  (the dot-dashed green contours) and the power spectrum divided by a ‘no-wiggles’ reference spectrum (the dotted blue contours). Degeneracy directions and likelihood contour levels are plotted as in Fig. 6.

level and bins spaced by  $20 h^{-1}$  Mpc are correlated at a level of  $\sim 55$  per cent.

## 5.2 Extraction of $D_V$

We determined the acoustic scale from the band-filtered correlation function by constructing a template function  $w_{0,\text{fid,galaxy}}(r)$  in the same style as Section 3.3 and then fitting the model:

$$w_{0,\text{mod}}(r) = b^2 w_{0,\text{fid,galaxy}}(\alpha r). \quad (21)$$

We determined the function  $w_{0,\text{fid,galaxy}}(r)$  by applying the transformation of equation (14) to the template galaxy correlation function  $\xi_{\text{fid,galaxy}}(s)$  defined in Section 3.3, as a function of  $\Omega_m h^2$ .

The best-fitting parameters to the band-filtered correlation function are  $\Omega_m h^2 = 0.130 \pm 0.011$ ,  $\alpha = 1.100 \pm 0.069$  and  $b^2 = 1.32 \pm 0.13$ , where the errors in each parameter are produced by marginalizing over the remaining two parameters. The minimum value of  $\chi^2$  is 10.5 for 14 degrees of freedom (17 bins minus three fitted parameters), indicating an acceptable fit to the data. Our measurement of the distortion parameter may be translated into a constraint on the distance scale  $D_V(z=0.6) = \alpha D_{V,\text{fid}} = 2279.2 \pm 142.4$  Mpc, corresponding to a 6.2 per cent measurement of the distance scale at  $z = 0.6$ . Probability contours of  $D_V(z=0.6)$  and  $\Omega_m h^2$  are overplotted in Fig. 14.

In Fig. 12 we compare the best-fitting band-filtered correlation function model to the WiggleZ data points (noting that the strong covariance between the data gives the misleading impression of a poor fit). We overplot a second model which corresponds to our best-fitting parameters but for which the no-wiggle reference power spectrum has been used in place of the full power spectrum. If we fit this no-wiggles model varying  $\Omega_m h^2$ ,  $\alpha$  and  $b^2$ , we find a minimum value of  $\chi^2 = 19.0$ , implying that the model containing acoustic features is favoured by  $\Delta\chi^2 = 8.5$ .

## 6 COMPARISON OF CLUSTERING STATISTICS

The distance-scale measurements of  $D_V(z = 0.6)$  using the four different clustering statistics applied in this paper are compared in Fig. 14. All four statistics give broadly consistent results for the measurement of  $D_V$  and  $\Omega_m h^2$ , with significant overlap between the respective 68 per cent confidence regions in this parameter space. This agreement suggests that systematic measurement errors in these statistics are not currently dominating the WiggleZ BAO fits.

There are some differences in detail between the results derived from the four statistics. Fits to the galaxy power spectrum are currently dominated by the power spectrum shape rather than the BAOs, such that the degeneracy direction lies along the line of constant apparent turnover scale. Fitting to only the ‘wiggles’ in Fourier space gives weaker constraints on the distance scale which (unsurprisingly) lie along the line of constant apparent BAO scale.

The correlation function and band-filtered correlation function yield very similar results (with the constraints from the standard correlation function being slightly stronger). Their degeneracy direction in the  $(\Omega_m h^2, D_V)$  parameter space lies between the two degeneracy directions previously mentioned, implying that both the BAO scale and correlation function shape are influencing the result. The slightly weaker constraint on the distance scale provided by the band-filtered correlation function compared to the standard correlation function is likely due to the suppression of information on small and large scales by the compensated filter, which is designed to reduce potential systematic errors in modelling the shape of the clustering pattern.

For our cosmological parameter fits in the remainder of the paper, we used the standard correlation function as our default choice of statistic. The correlation function provides the tightest measurement of the distance scale from our current data set and encodes the most significant detection of the BAO signal.

## 7 DISTILLED PARAMETERS

For each of the clustering statistics determined above, the measurement of  $D_V$  is significantly correlated with the matter density  $\Omega_m h^2$  which controls both the shape of the clustering pattern and the length-scale of the standard ruler (see Fig. 14). It is therefore useful to recast these BAO measurements in a manner less correlated with  $\Omega_m h^2$  and more representative of the observable combination of parameters constrained by the BAOs. These ‘distilled parameters’ are introduced and measured in this section.

### 7.1 CMB information

The length-scale of the BAO standard ruler and shape of the linear clustering pattern are calibrated by CMB data. The cosmological information contained in the CMB may be conveniently encapsulated by the *WMAP* ‘distance priors’ (Komatsu et al. 2009). We use the 7-year *WMAP* results quoted in Komatsu et al. (2011).

First, the CMB accurately measures the characteristic angular scale of the acoustic peaks  $\theta_A \equiv r_s(z_*)/(1+z_*)D_A(z_*)$ , where  $r_s(z_*)$  is the size of the sound horizon at last scattering and  $D_A(z_*)$  is the physical angular diameter distance to the decoupling surface. This quantity is conventionally expressed as a characteristic acoustic index:

$$\ell_A \equiv \pi/\theta_A = \pi(1+z_*) \frac{D_A(z_*)}{r_s(z_*)} = 302.09 \pm 0.76. \quad (22)$$

The complete CMB likelihood is well reproduced by combining this measurement of  $\ell_A$  with the ‘shift parameter’ defined by

$$\mathcal{R} \equiv \frac{\sqrt{\Omega_m H_0^2}}{c} (1+z_*) D_A(z_*) = 1.725 \pm 0.018, \quad (23)$$

and the redshift of recombination (using the fitting function given as equations 66–68 in Komatsu et al. 2009)

$$z_* = 1091.3 \pm 0.91. \quad (24)$$

The inverse covariance matrix for  $(\ell_A, \mathcal{R}, z_*)$  is given as table 10 in Komatsu et al. (2011) and is included in our cosmological parameter fit.

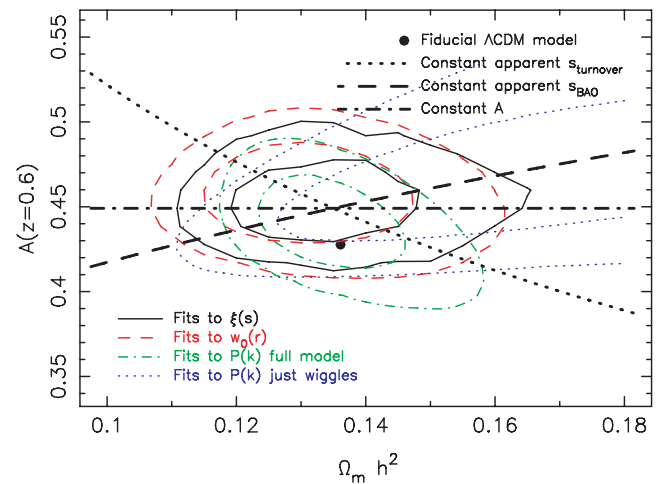
### 7.2 Measuring $A(z)$

As noted in Eisenstein et al. (2005) and discussed in Section 3.4 above, the parameter combination

$$A(z) \equiv \frac{D_V(z) \sqrt{\Omega_m H_0^2}}{cz}, \quad (25)$$

which we refer to as the ‘acoustic parameter’, is particularly well constrained by distance fits which utilize a combination of acoustic oscillation and clustering shape information, since in this situation the degeneracy direction of constant  $A(z)$  lies approximately perpendicular to the minor axis of the measured  $(D_V, \Omega_m h^2)$  probability contours. Conveniently,  $A(z)$  is also independent of  $H_0$  (given that  $D_V \propto 1/H_0$ ). Fig. 15 displays the measurements resulting from fitting the parameter set  $(A, \Omega_m h^2, b^2)$  to the four WiggleZ clustering statistics and marginalizing over  $b^2$ . The results of the parameter fits are displayed in Table 1; the correlation function yields  $A(z = 0.6) = 0.452 \pm 0.018$  (i.e. with a measurement precision of 4.0 per cent). For this clustering statistic in particular, the correlation between measurements of  $A(z)$  and  $\Omega_m h^2$  is very low. Given that the CMB provides a very accurate determination of  $\Omega_m h^2$  (via the distance priors), we do not use the WiggleZ determination of  $\Omega_m h^2$  in our cosmological parameter fits, but just use the marginalized measurement of  $A(z)$ .

The measurement of  $A(z)$  involves the assumption of a model for the shape of the power spectrum, which we parametrize by  $\Omega_m h^2$ . Essentially the full power spectrum shape, rather than just the BAOs, is being used as a standard ruler, although the two features combine



**Figure 15.** A comparison of the results of fitting different WiggleZ clustering measurements in the same style as Fig. 14, except that we now fit for the parameter  $A(z = 0.6)$  (defined by equation 25) rather than  $D_V(z = 0.6)$ .



in such a way that  $A$  and  $\Omega_m h^2$  are uncorrelated. However, given that a model for the full power spectrum is being employed, we refer to these results as ‘LSS’ rather than BAO constraints, where appropriate.

### 7.3 Measuring $d_z$

In the case of a measurement of the BAOs in which the shape of the clustering pattern is marginalized over, the  $(D_V, \Omega_m h^2)$  probability contours would lie along a line of constant apparent BAO scale. Hence the extracted distances are measured in units of the standard-ruler scale, which may be conveniently quoted using the distilled parameter  $d_z \equiv r_s(z_d)/D_V(z)$ , where  $r_s(z_d)$  is the comoving sound horizon size at the baryon drag epoch. In contrast to the acoustic parameter,  $d_z$  provides a purely geometric distance measurement that does not depend on knowledge of the power spectrum shape. The information required to compare the observations to theoretical predictions also varies between these first two distilled parameters: the prediction of  $d_z$  requires prior information about  $h$  (or  $\Omega_m h^2$ ), whereas the prediction of  $A(z)$  does not. We also fitted the parameter set  $(d_{0.6}, \Omega_m h^2, b^2)$  to the four WiggleZ clustering statistics. The results of the parameter fits are displayed in Table 1; the correlation function yields  $d_{0.6} = 0.0692 \pm 0.0033$  (i.e. with a measurement precision of 4.8 per cent). We note that, given the WiggleZ fits are in part driven by the shape of the power spectrum as well as the BAOs, there is a weak residual correlation between  $d_{0.6}$  and  $\Omega_m h^2$ .

When calculating the theoretical prediction for this parameter, we obtained the value of  $r_s(z_d)$  for each cosmological model tested using equation (6) of Eisenstein & Hu (1998), which is a fitting formula for  $r_s(z_d)$  in terms of the values of  $\Omega_m h^2$  and  $\Omega_b h^2$ . In our analysis, we fixed  $\Omega_b h^2 = 0.0226$  which is consistent with the measured CMB value (Komatsu et al. 2009); we find that marginalizing over the uncertainty in this value does not change the results of our cosmological analysis.

We note that  $r_s(z_d)$  is determined from the matter and baryon densities in units of Mpc (not  $h^{-1}$  Mpc), and thus a fiducial value of  $h$  must also be used when determining  $d_z$  from data (we chose  $h = 0.71$ ). However, the quoted observational result  $d_{0.6} = 0.0692 \pm 0.0033$  is actually independent of  $h$ . Adoption of a different value of  $h$  would result in a shifted standard-ruler scale (in units of  $h^{-1}$  Mpc) and hence shifted best-fitting values of  $\alpha$  and  $D_V$  in such a way that  $d_z$  is unchanged. However, although the observed value of  $d_z$  is independent of the fiducial value of  $h$ , the model fitted to the data still depends on  $h$  as remarked above.

### 7.4 Measuring $R_z$

The measurement of  $d_z$  may be equivalently expressed as a ratio of the low-redshift distance  $D_V(z)$  to the distance to the last-scattering surface, exploiting the accurate measurement of  $\ell_A$  provided by the CMB. We note that the value of  $R_z$  depends on the behaviour of dark energy between redshift  $z$  and recombination, whereas a constraint derived from  $d_z$  only depends on the properties of dark energy at redshifts lower than  $z$ . Taking the product of  $d_z$  and  $\ell_A/\pi$  approximately cancels out the dependence on the sound horizon scale:

$$\begin{aligned} 1/R_z &\equiv \ell_A d_z / \pi = (1 + z_*) \frac{D_A(z_*)}{r_s(z_*)} \frac{r_s(z_d)}{D_V(z)} \\ &\approx (1 + z_*) \frac{D_A(z_*)}{D_V(z)} \times 1.044. \end{aligned} \quad (26)$$

The value 1.044 is the ratio between the sound horizon at last scattering and at the baryon drag epoch. Although this is a model-dependent quantity, the change in redshift between recombination and the end of the drag epoch is driven by the relative number density of photons and baryons, which is a feature that does not change much across the range of viable cosmological models. Combining our measurement of  $d_{0.6} = 0.0692 \pm 0.0033$  from the WiggleZ correlation function fit with  $\ell_A = 302.09 \pm 0.76$  (Komatsu et al. 2011), we obtain  $1/R_{0.6} = 6.65 \pm 0.32$ .

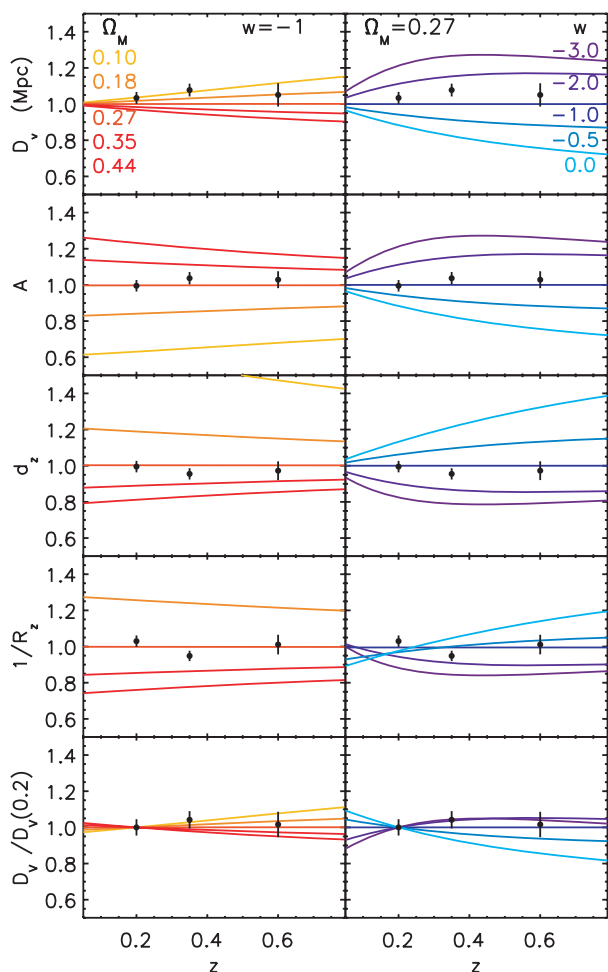
### 7.5 Measuring distance ratios

Finally, we can avoid the need to combine the BAO fits with CMB measurements by considering distance ratios between the different redshifts at which BAO detections have been performed. Measurements of  $D_V(z)$  alone are dependent on the fiducial cosmological model and assumed standard-ruler scale: an efficient way to measure  $D_V(z_2)/D_V(z_1)$  is by calculating  $d_{z_1}/d_{z_2}$ , which is independent of the value of  $r_s(z_d)$ . Percival et al. (2010) reported BAO fits to the SDSS LRG sample in two correlated redshift bins  $d_{0.2} = 0.1905 \pm 0.0061$  and  $d_{0.35} = 0.1097 \pm 0.0036$  (with correlation coefficient 0.337). Ratioing  $d_{0.2}$  with the independent measurement of  $d_{0.6} = 0.0692 \pm 0.0033$  from the WiggleZ correlation function and combining the errors in quadrature, we find that  $D_V(0.6)/D_V(0.2) = 2.753 \pm 0.158$ . Percival et al. (2010) report  $D_V(0.35)/D_V(0.2) = 1.737 \pm 0.065$  [where in this latter case the error is slightly tighter than obtained by adding errors in quadrature because of the correlation between  $D_V(0.2)$  and  $D_V(0.35)$ ]. These two distance ratio measurements are also correlated by the common presence of  $D_V(0.2)$  in the denominators; the correlation coefficient is 0.313.

### 7.6 Comparison of distilled parameters

Fig. 16 compares the measurements of the distilled parameters introduced in this section, using both WiggleZ and LRG data, to a series of cosmological models varying either  $\Omega_m$  or the dark energy equation of state,  $w$ , relative to a fiducial model with  $\Omega_m = 0.27$  and  $w = -1$ . These plots have all been normalized to the fiducial model and plotted on the same scale so that the level of information contained in the different distilled parameters can be compared. For presentational purposes in Fig. 16, we converted the Percival et al. (2010) measurements of  $d_z$  into constraints on the acoustic parameter  $A(z)$ , using their fiducial values of  $r_s(z_d) = 154.7$  Mpc and  $\Omega_m h^2 = 0.1296$  and employing the same fractional error in the distilled parameter.

We note that  $A(z)$  and the ratio of  $D_V(z)$  values are the only distilled parameters that are independent of CMB data: the measurement of  $1/R_z$  uses  $\ell_A$  from the CMB, and whilst the observed value of  $d_z$  is independent of the CMB, we need to marginalize over  $\Omega_m h^2$  or  $h$  in order to compare it with theoretical models. Fig. 17 illustrates the differing information encapsulated by each of these distilled parameters by plotting likelihood contours of  $(\Omega_m, \Omega_\Lambda)$  for  $w = -1$  and  $(\Omega_m, w)$  for  $\Omega_k = 0$  fitted to these data. The significant differences in the resulting likelihood contours exemplify the fact that in some cases we have not used all the information in the galaxy data [e.g.  $D_V(z)/D_V(0.2)$  uses only the ratio of BAO scales, neglecting information from the absolute scale of the standard ruler], while in other cases we have already included significant information from the CMB (e.g.  $1/R_z$ ). This illustrates that we must be careful to include additional CMB data in a self-consistent manner, not double-counting the information. This plot also serves to illustrate the mild tension between the BAO and CMB results, which can



**Figure 16.** Measurements of the distilled BAO parameters extracted from WiggleZ and LRG data sets as a function of redshift. From upper to lower, we plot  $D_V$  (in Mpc), the acoustic parameter  $A(z)$ ,  $d_z \equiv r_s(z_d)/D_V(z)$ ,  $1/R_z \equiv \ell_A d_z/\pi$  and  $D_V(z)/D_V(z=0.2)$ . The left-hand column shows a set of different cosmological models varying  $\Omega_m$  with  $[\Omega_k, w] = [0, -1]$  in each case. The right-hand column displays a range of models varying  $w$  with  $[\Omega_m, \Omega_k] = [0.27, 0]$  in each case.

be seen from the fact that the  $1/R_z$  constraints are offset from the centre of the  $A(z)$  constraints. The WiggleZ and LRG distilled BAO parameters used in these fits are summarized for convenience in Table 2.

## 8 COSMOLOGICAL PARAMETER MEASUREMENTS

In this section we present cosmological parameter fits to the distilled BAO parameters measured above. We consider two versions of the standard cosmological model. The first version is the standard  $\Lambda$ CDM model in which dark energy is a cosmological constant with equation of state  $w = -1$  and spatial curvature is a free parameter; we fit for  $\Omega_m$  and the cosmological constant density  $\Omega_\Lambda$ . The second version is the flat  $w$ CDM model in which spatial curvature is fixed at  $\Omega_k = 0$  but the equation of state of dark energy is a free parameter; we fit for  $\Omega_m$  and  $w$ .

Unless otherwise stated, we fitted these cosmological models to the WiggleZ measurement of  $A(z = 0.6)$  combined with the measurements of  $d_z$  from LRG samples at  $z = 0.2$  and  $0.35$  (Percival

et al. 2010). The distilled parameter  $A(z)$  (measured from the galaxy correlation function) is the most appropriate choice for quantifying the WiggleZ BAO measurement because it is uncorrelated with  $\Omega_m h^2$ , as demonstrated by Fig. 15. The parameter  $d_z$  provides the best representation of the Percival et al. (2010) BAO data, because the shape of the clustering pattern was marginalized over in that analysis.

### 8.1 BAO alone

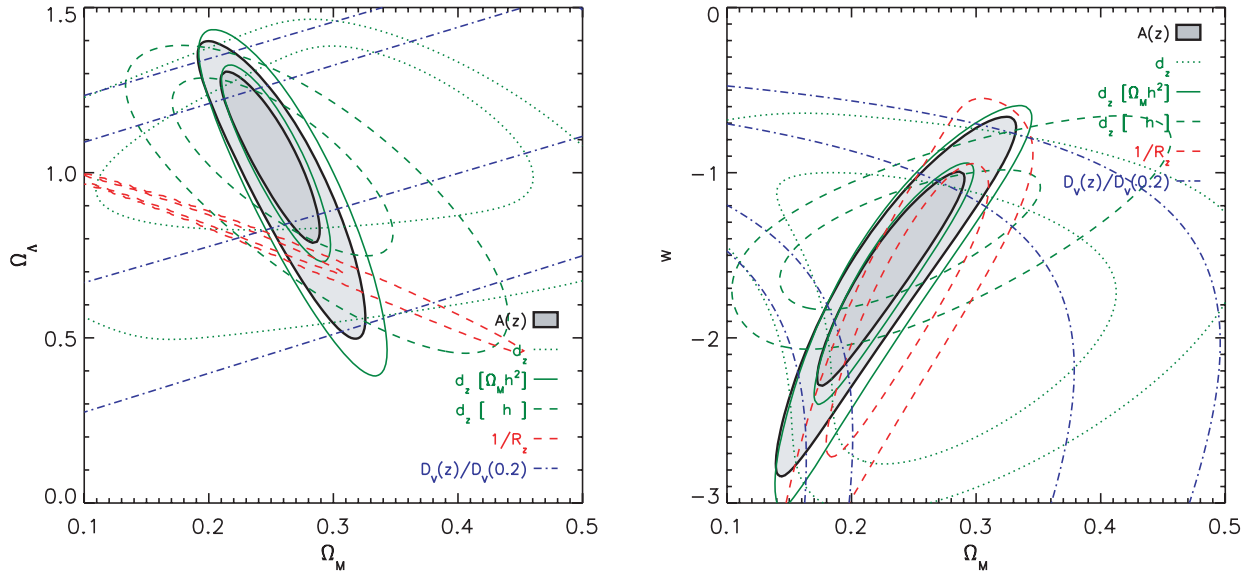
As a first step in the cosmological analysis, we considered the constraints on cosmological parameters obtained using BAO data alone. Two of the distilled parameters allow the derivation of cosmological constraints based on only BAO measurements: the ratio of  $D_V(z)$  measurements and the acoustic parameter,  $A(z)$ .

Ratios of  $D_V(z)$  measurements are of particular interest because they provide constraints on the cosmic expansion history using geometric information alone, independent of the shape of the clustering pattern and the absolute scale of the standard ruler. Plots of the resulting cosmological constraints in our two test models ( $\Lambda$ CDM and  $w$ CDM) are shown in Fig. 18. Using only LRG BAO data, the single available distance ratio  $D_V(0.35)/D_V(0.2)$  provided relatively weak constraints on cosmological parameters and, in particular, could not confirm the acceleration of the expansion of the universe. The addition of the second distance ratio based on WiggleZ data,  $D_V(0.6)/D_V(0.2)$ , significantly improves these constraints. For the first time, purely geometric distance ratios from BAO measurements demonstrate that the cosmic expansion is accelerating: assuming the flat  $w$ CDM model, a dark energy fluid with  $w < -1/3$  is required with a likelihood of 99.8 per cent (assuming the flat prior  $-3 < w < 0$ ).

The improvement in the  $\chi^2$  statistic comparing the best-fitting  $\Lambda$ CDM model ( $\Omega_m, \Omega_\Lambda$ ) = (0.25, 1.1) to the Einstein de-Sitter model ( $\Omega_m, \Omega_\Lambda$ ) = (1.0, 0.0) is  $\Delta\chi^2 = 18$ , whilst comparing the best-fitting  $\Lambda$ CDM model to the open CDM model ( $\Omega_m, \Omega_\Lambda$ ) = (0.27, 0.0), we obtain  $\Delta\chi^2 = 8$ . Even given the extra parameter in the  $\Lambda$ CDM model, information criteria tests consider this level of improvement in  $\chi^2$  to be significant evidence in favour of the  $\Lambda$ CDM model compared to a model with no dark energy.

Following the addition of the WiggleZ measurement, the BAO data alone require accelerating cosmic expansion with a higher level of statistical confidence than the initial luminosity distance measurements from supernovae (SNe) that are considered to be the first direct evidence of accelerating expansion (compare the left-hand panel of Fig. 18 with fig. 6 of Riess et al. 1998). Although these BAO measurements are not yet competitive with the latest SN constraints, it is nevertheless reassuring that a standard-ruler measurement of the expansion of the universe, subject to an entirely different set of potential systematic uncertainties, produces a result in agreement with the standard-candle measurement.

The cosmological constraints from the acoustic parameter  $A(z)$  are much more constraining than the  $D_V(z)$  ratios because they implicitly incorporate a model for the clustering pattern and standard-ruler scale as a function of  $\Omega_m h^2$ . Fig. 19 displays the resulting cosmological parameter fits to the WiggleZ measurement of  $A(z = 0.6)$  combined with the LRG measurements of  $d_z$  at  $z = 0.2, 0.35$ . For the purposes of this figure we combine the  $d_z$  measurements with a prior  $\Omega_m h^2 = 0.1326 \pm 0.0063$  (following Percival et al. 2010); in the next subsection we use the *WMAP* distance priors instead. The improvement delivered by the WiggleZ data can be seen by comparing the shaded grey contour, which is the combination of the LRG results, with the solid black contours representing the



**Figure 17.** Likelihood contours ( $1\sigma$  and  $2\sigma$ ) derived from model fits to the different distilled parameters which may be used to encapsulate the BAO results from LRG and WiggleZ data. In each case, the contours represent the combination of the three redshift bins for which we have BAO data,  $z = [0.2, 0.35, 0.6]$ . The thick solid lines with grey shading show the  $A(z)$  parameter constraints, which are the most appropriate representation of WiggleZ data. The three sets of green lines show constraints from  $d_z$  assuming three different priors: green solid lines include a prior  $\Omega_m h^2 = 0.1326 \pm 0.0063$  (Komatsu et al. 2009, as used by Percival et al. 2010); green dotted lines marginalize over a flat prior of  $0.5 < h < 1.0$  and green dashed lines marginalize over a Gaussian prior of  $h = 0.72 \pm 0.03$ . Red dashed lines show the  $1/R_z$  constraints, whilst fits to the ratios of BAO distances  $D_V(z)/D_V(0.2)$  are shown by blue dot-dashed lines. The left-hand panel shows results for curved cosmological-constant universes parametrized by  $(\Omega_m, \Omega_\Lambda)$ , and the right-hand panel displays results for flat dark-energy universes parametrized by  $(\Omega_m, w)$ . Comparisons between these contours reveal the differing levels of information encoded in each distilled parameter. By combining each type of distilled parameter with the CMB data in a correct manner, self-consistent results should be achieved.

**Table 2.** Measurements of the distilled BAO parameters at redshifts  $z = 0.2, 0.35$  and  $0.6$  from LRG and WiggleZ data, which are used in our cosmological parameter constraints. The LRG measurements at  $z = 0.2$  and  $0.35$  are correlated with coefficient  $0.337$  (Percival et al. 2010). The two distance ratios  $d_{0.2}/d_{0.35}$  and  $d_{0.2}/d_{0.6}$  are correlated with coefficient  $0.313$ . The different measurements of  $1/R_z$  are correlated by the common presence of the  $\ell_A$  variable and by the covariance between  $d_{0.2}$  and  $d_{0.35}$ . Our default cosmological fits use the WiggleZ measurement of  $A(z = 0.6)$  combined with the LRG measurements of  $d_{0.2}$  and  $d_{0.35}$ , indicated by bold font.

	$A(z)$ measured	$d_z$ measured	$D_V(z)/D_V(0.2)$ $= d_{0.2}/d_z$	$1/R_z$ $= \ell_A d_z / \pi$
$z = 0.2$ (LRG)	$0.488 \pm 0.016$	<b><math>0.1905 \pm 0.0061</math></b>	–	$18.32 \pm 0.59$
$z = 0.35$ (LRG)	$0.484 \pm 0.016$	<b><math>0.1097 \pm 0.0036</math></b>	$1.737 \pm 0.065$	$10.55 \pm 0.35$
$z = 0.6$ (WiggleZ)	<b><math>0.452 \pm 0.018</math></b>	$0.0692 \pm 0.0033$	$2.753 \pm 0.158$	$6.65 \pm 0.32$

total BAO constraint including the new WiggleZ data. The current WiggleZ data set delivers an improvement of about 50 per cent in the measurement of  $\Omega_\Lambda$  and about 30 per cent in the measurement of  $w$ , based on LSS data alone. The marginalized parameter measurements are  $\Omega_m = 0.25^{+0.05}_{-0.04}$  and  $\Omega_\Lambda = 1.1^{+0.2}_{-0.4}$  (for  $\Lambda$ CDM) and  $\Omega_m = 0.23 \pm 0.06$  and  $w = -1.6^{+0.6}_{-0.7}$  (for  $w$ CDM).

The BAO results continue to prefer a more negative dark energy equation of state or a higher cosmological constant density than the CMB or SNe data. We explore this further in the following section.

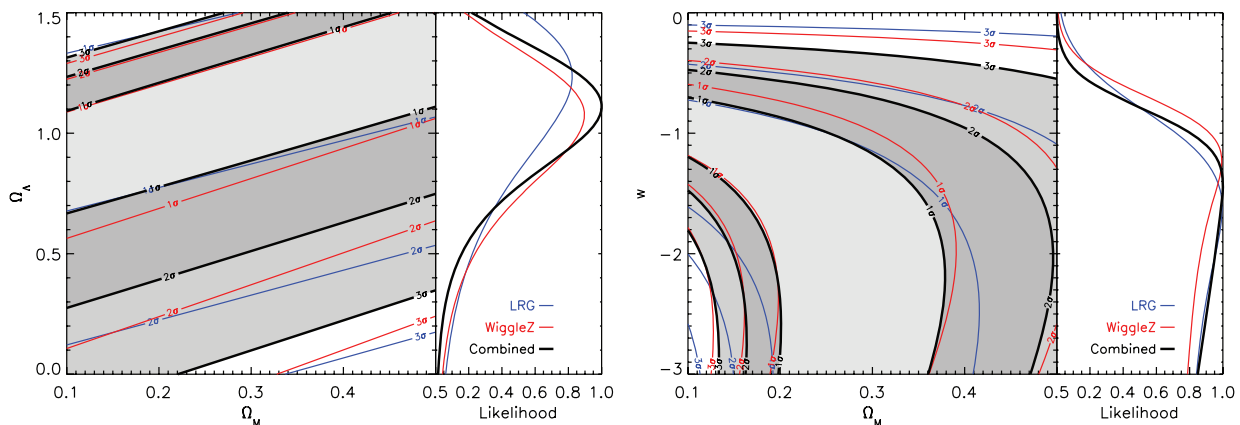
## 8.2 BAO combined with CMB and SNe

We now combine these LSS measurements with other cosmological data sets. We incorporated the CMB data using the *WMAP* distance priors in  $(\ell_A, \mathcal{R}, z_*)$  described in Section 7.1. We fitted a model parametrized by  $(\Omega_m, w, h)$  using flat priors  $0.1 < \Omega_m < 0.5$ ,  $0.5 < h < 1.0$  and  $-3 < w < 0$  and assuming a flat Universe ( $\Omega_k = 0$ ). Fig. 20 compares the combined LSS cosmological parameter mea-

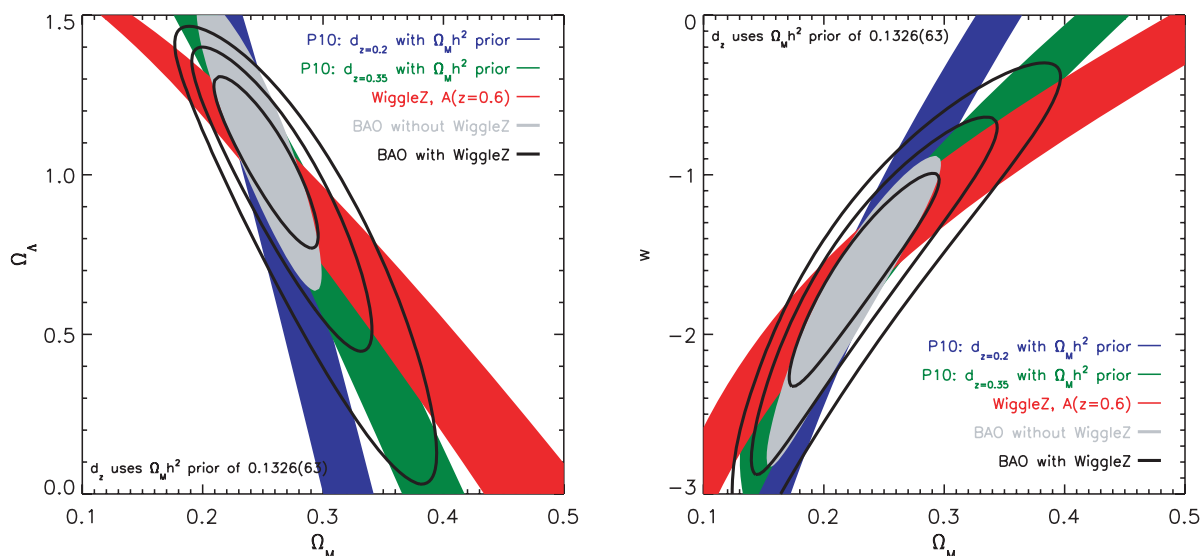
surements to the CMB constraints in both the  $(\Omega_m, w)$  and  $(h, w)$  planes, marginalizing over  $h$  and  $\Omega_m$ , respectively. The marginalized measurements of each parameter are  $\Omega_m = 0.287^{+0.029}_{-0.028}$ ,  $w = -0.982^{+0.154}_{-0.189}$  and  $h = 0.692^{+0.044}_{-0.038}$ .

We also combined these constraints with those arising from Type Ia SNe data from the ‘Union2’ compilation by Amanullah et al. (2010), which includes data from Hamuy et al. (1996), Riess et al. (1999, 2007), Astier et al. (2006), Jha et al. (2006), Wood-Vasey et al. (2007), Holtzman et al. (2008), Hicken et al. (2009) and Kessler et al. (2009). Using all of these data sets (LSS+CMB+SNe) the best-fitting  $w$ CDM model is  $(\Omega_m, w) = (0.284 \pm 0.016, -1.026 \pm 0.081)$ . This model provides a good fit to the data with a minimum  $\chi^2$  per degree of freedom of 0.95.

The best-fitting parameter values based on LSS data alone are offset by slightly more than one standard deviation from the best-fitting parameter values of the combined LSS+CMB+SN fit. This mild tension between LSS and CMB+SN results was already evident in the Percival et al. (2007) measurement of BAOs in the



**Figure 18.** Likelihood contours for cosmological parameter fits to BAO measurements using  $D_V(z)$  distance ratios. We fit two different models: curved cosmological-constant universes parametrized by  $(\Omega_m, \Omega_\Lambda)$  and flat dark-energy universes parametrized by  $(\Omega_m, w)$ . Blue contours show the constraints using the measurement of  $D_V(0.35)/D_V(0.2)$  obtained by Percival et al. (2010). Red contours display the new constraints in  $D_V(0.6)/D_V(0.2)$  derived using WiggleZ data. The combination of these measurements is plotted as the grey shaded contours. 1D marginalized likelihoods for  $\Omega_\Lambda$  and  $w$  are displayed on the right-hand side of each contour plot. In the flat  $w$ CDM model, the BAO distance ratios alone require accelerating expansion ( $w < -1/3$ ) with a likelihood of 99.8 per cent. Compared to the LRG data, WiggleZ does not favour as high values of  $\Omega_\Lambda$  or as negative values of  $w$ .



**Figure 19.** A comparison of the WiggleZ results and previous BAO measurements. The blue and green solid contours show the redshift  $z = 0.2$  and  $0.35$  bins from Percival et al. (2010), using the  $d_z$  parameter and including a CMB-motivated prior  $\Omega_m h^2 = 0.1326 \pm 0.0063$ . The red contour displays the fit to the WiggleZ measurement of  $A(z)$  at  $z = 0.6$ , which is independent of any prior in  $\Omega_m h^2$ . We note that the degeneracy directions in these plots rotate with redshift, demonstrating the utility of combining measurements at different redshifts. The grey shaded region shows the combination of  $d_{0.2}$  and  $d_{0.35}$  (including the correlation). The black solid lines are the  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  contours of the final result combining all these BAO measurements. The addition of WiggleZ data reduces the  $1\sigma$  uncertainty by about 50 per cent in  $\Omega_\Lambda$  and about 30 per cent in  $w$ , particularly disfavouring high values of  $\Omega_\Lambda$  and more negative values of  $w$ . We note that in our final cosmological measurements, we do not use a prior in  $\Omega_m h^2$  but instead combine our results with the *WMAP* distance priors.

SDSS; the LSS measurements prefer a slightly higher rate of acceleration or a more negative equation of state of dark energy. The new WiggleZ results reported here serve to amplify that tension, although the level of discrepancy is not statistically significant.

In order to investigate whether more complex models give a better statistical fit to these data sets, we performed fits of two additional cosmological models. The first is a  $w$ CDM model in which we allow curvature to be a free parameter; i.e. we fit for  $\Omega_m$ ,  $\Omega_k$  and  $w$ . The second is a  $w(a)$ CDM model in which the equation of state of dark energy is allowed to vary linearly with scale-factor (Chevallier–Polarski–Linder parametrization; Chevallier & Polarski 2001; Linder 2003); i.e. we fit for  $\Omega_m$ ,  $w_0$  and  $w_a$ . If these additional parameters are justified we should find that the minimum

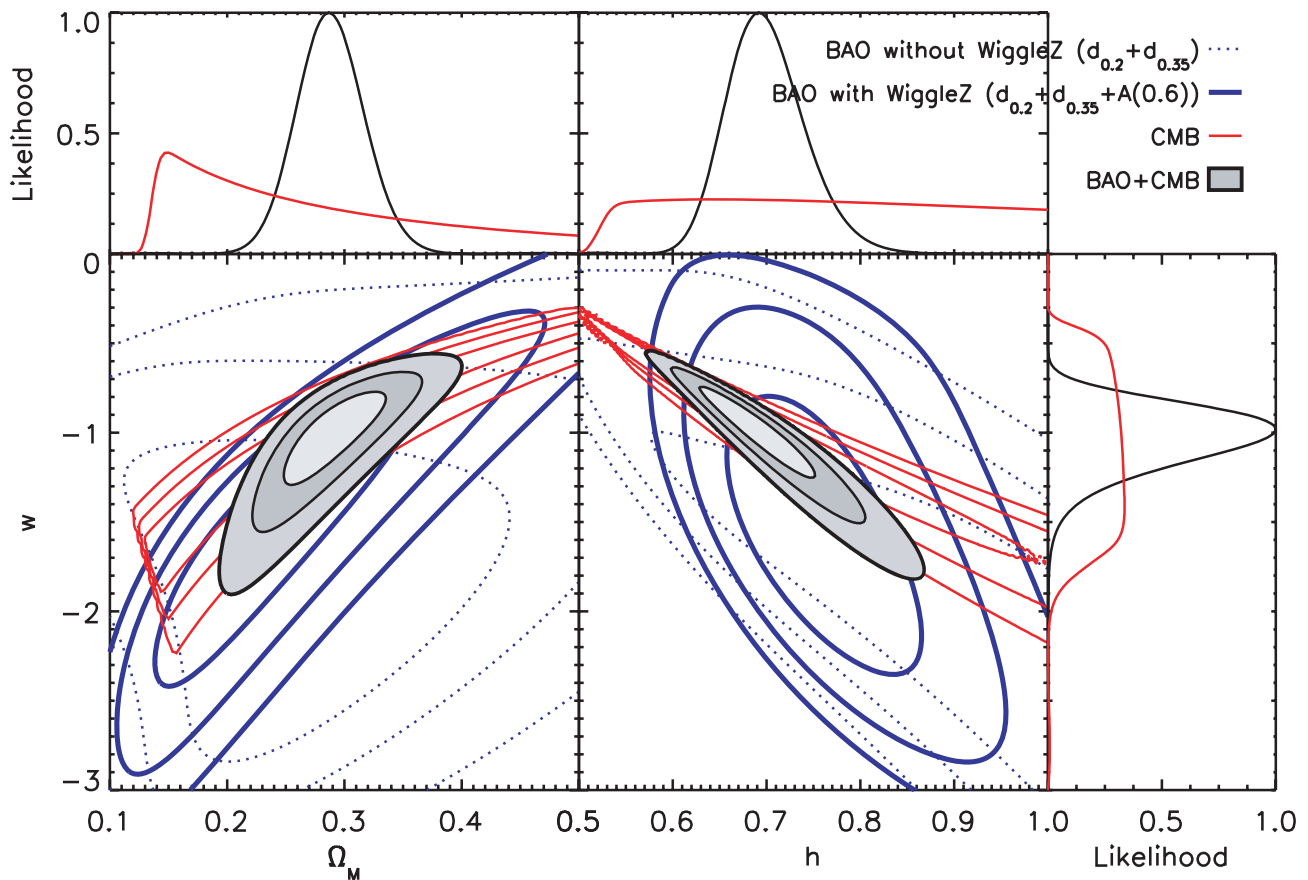
value of the  $\chi^2$  statistic decreases significantly. However, we in fact find that the addition of the extra degrees of freedom improves the  $\chi^2$  value of the best-fitting model by at most  $\Delta\chi^2 = 0.6$  compared to the  $\Lambda$ CDM model, and significantly degrades the errors in our fitted parameters. In terms of information criteria, this does not represent a sufficient improvement to justify the addition of the extra degrees of freedom.

## 9 CONCLUSIONS

We summarize our conclusions as follows.

- (i) This intermediate sample of the WiggleZ Dark Energy Survey, containing  $N = 132\,509$  galaxies, permits a convincing detection





**Figure 20.** Likelihood contours for cosmological parameter fits to LRG and WiggleZ BAO data, compared to and combined with CMB measurements. We have fitted for three parameters ( $\Omega_m$ ,  $w$  and  $h$ ) assuming a flat Universe. The left-hand panel of contours marginalizes over  $h$  and the right-hand panel of contours marginalizes over  $\Omega_m$ . The LSS constraints appear in blue, as dotted lines showing the Percival et al. (2010) results for  $d_z$  with no CMB prior, and as solid lines combining that data with the WiggleZ measurement of  $A(z)$ . The red contours show the CMB results (using the *WMAP* distance priors). The grey shaded contours display the combination of BAO and CMB measurements. 1D likelihood distributions for each of the parameters are shown as the insets. The BAO constraints shown here are much wider than those plotted in Fig. 19, because they do not assume any prior in  $\Omega_m h^2$ . The improvement contributed by the BAO, as compared to the CMB constraints alone, can be seen by comparing the black likelihood distributions, representing the total constraints, with the red likelihood distributions, which are for the CMB alone. (The sharp edges of the CMB likelihood distributions arise due to the flat priors  $0.5 < h < 1.0$  and  $0.1 < \Omega_m < 0.5$  we adopted).

of BAOs at the highest redshift achieved to date,  $z = 0.6$ . We have quantified the baryon oscillations in both the galaxy correlation function and power spectrum statistics. The statistical significance of the correlation function detection exceeds  $3\sigma$ .

(ii) We present the first measurement from a galaxy survey of the band-filtered BAO estimator  $w_0(r)$  defined by Xu et al. (2010), which also exhibits strong evidence of BAOs, and we suggest some improvements in its implementation. The distance-scale measurements resulting from  $w_0(r)$  are slightly less precise than those obtained by fits to the standard correlation function, likely due to the suppression by the filtering function of clustering information on small and large scales.

(iii) The clustering statistics are well fitted by a non-linear power spectrum model including a Gaussian damping factor which models coherent flows on  $100 h^{-1}$  Mpc scales. We show that our results are not sensitive to the details of the quasi-linear model by comparing five different implementations suggested in the literature. We use the GigggleZ  $N$ -body simulation to demonstrate that scale-dependent bias effects in the WiggleZ galaxy distribution have an amplitude of 1 per cent on  $10 h^{-1}$  Mpc scales, compared to 10 per cent for more highly biased LRGs.

(iv) We measured the distance–redshift relation  $D_V(z = 0.6)$  with an accuracy of about 5 per cent using the scale dilation method. The self-consistency of the measurement amongst the independent clustering statistics suggests that systematic measurement errors are not dominating these results. Our data best constrain the quantity  $A(z) = D_V(z)\sqrt{\Omega_m H_0^2}/cz$ , which lies perpendicular to the principal degeneracy direction of our contours and which is measured with an accuracy of 4 per cent: we find  $A(z = 0.6) = 0.452 \pm 0.018$ .

(v) The distance ratios between our measurements and previous analyses of the LRG distribution at redshifts  $z = 0.2$  and  $0.35$  are consistent with a flat  $\Lambda$ CDM cosmological model which also provides a good fit to the CMB distance priors. Addition of the WiggleZ data allows us to establish, using geometric distance ratios alone, that the equation of state of dark energy drives an accelerating expansion ( $w < -1/3$ ) with 99.8 per cent likelihood, assuming a flat prior  $-3 < w < 0$ . The current WiggleZ data set delivers an improvement of about 50 per cent in the measurement of  $\Omega_\Lambda$  and 30 per cent in the measurement of  $w$ , based on BAO data alone. The WiggleZ measurement confirms the mild tension that was previously reported between CMB and BAO measurements of

the acceleration of the Universe, whereby the BAO data favour a slightly higher acceleration rate.

(vi) Cosmological parameter fits using BAO and CMB data are consistent with those based on current SNe data; this cross-check does not yield evidence for systematic errors. Combining all current BAO, CMB and SNe data, we find that the best-fitting  $w$ CDM model is  $(\Omega_m, w) = (0.284 \pm 0.016, -1.026 \pm 0.081)$ . These data do not justify the addition of another degree of freedom such as non-zero spatial curvature or an evolving equation of state of dark energy.

When the complete BAO data set from the WiggleZ survey is available, we plan to split our data into redshift bins and explore fitting for the BAOs separately in the tangential and radial directions, improving the cosmological constraints presented in this paper.

The WiggleZ data set enables further tests of the cosmological model, complementary to those involving BAOs. Two such results will be published at a similar time as the current paper. First, we have used redshift-space distortions in the WiggleZ sample to measure accurately the growth rate of structure in the redshift range  $0.1 < z < 0.9$ , finding that this is consistent with the predictions of the  $\Lambda$ CDM model (Blake et al. 2011a). Secondly, we have used the Alcock–Paczynski test to perform non-parametric reconstructions of the cosmic expansion history (Blake et al. 2011b).

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